



**Department of AERONAUTICS and ASTRONAUTICS
STANFORD UNIVERSITY**

Final Report

**NASA Grant NAG3-2284
Development of High Regression Rate
Hybrid-Combustion Stability**

Submitted to the

**NASA Glenn Research Center
Space Systems and Grants Branch
2100 Brookpark Road, MS 86-8
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Principal Investigator

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REPORT ON HYBRID ROCKET COLD FLOW EXPERIMENTS

Student researcher - Siina Haapanen

Experimental work in collaboration with Greg Zilliac, computational results by Lyndell King

Work supported by NASA Grant NAG3 - 2884

Introduction

The discovery of paraffin based fuels has lead to renewed interest in hybrid rocket research. Experiments have shown that they burn 3-5 times faster than conventional hybrid fuels. High thrust level that would have required a multi-port design in the past can now be achieved with a single-port motor. While tests performed in Stanford and NASA Ames have demonstrated the paraffin hybrids to be a promising technology, one of the major challenges has been the relatively low efficiency. The c^* efficiency has ranged between 80% and 90% in experiments conducted at the Ames Hybrid Combustion Facility (HCF). The test motor in these experiments had a 45 inch long fuel grain with the initial port diameter ranging between 3 and 5 _ inches. The c^* efficiency is defined as the ratio of measured and theoretical characteristic velocities and is related to how completely the fuel and oxidizer are converted to combustion products. A low efficiency means that the reactants burn incompletely, and the reaction does not release the maximum possible amount of energy.

Two species react incompletely if either their chemical reaction rate is slow or if they are not mixed adequately. Chemical reactions take place through molecular collisions, and if the two species do not come into contact because of insufficient mixing, they can not form products. Typical fuels and oxidizers exhibit fast reaction rates. The characteristic time scale of chemical reactions is much smaller than the time scales related to convection and diffusion. In other words, the Damk hler number, $Da = \tau_{phys} / \tau_{chem}$, where τ_{phys} and τ_{chem} are characteristic physical and chemical time scales, is large. In this limit of large Da , chemical kinetics is not thought to influence the efficiency. Therefore, a high efficiency can be accomplished by effective mixing. It can be challenging to achieve good mixing between the reactants in a hybrid motor. Fuel and oxidizer are stored separately and in different phases. The solid fuel must vaporize and the reactants must diffuse to meet each other before chemical reactions can take place. In contrast, the fuel and oxidizer in a solid rocket are premixed, and in liquid-liquid systems, the reactants can be injected into the combustion chamber in a way that promotes mixing. Consequently, hybrids are generally less efficient than solid or liquid systems. It should be noted, however, that while storing the reactants separately makes it difficult achieve high efficiency, the same feature also makes hybrid rockets inherently safe which is one of their major advantages over other types of rockets.

The efficiency of a hybrid rocket is generally found to increase with the length of the motor. The residence time increases with the length of the motor, allowing more time for the reactants to diffuse towards each other. However, simply making a motor longer to increase its efficiency is not a practical solution, and alternate approaches must be

considered. The focus of the present research is to find a way to mix the fuel and oxidizer better that is practical and simple to implement in a hybrid motor. The approach taken is to alter the flow field by generating disturbances to the oxidizer flow upstream of the combustion chamber, and to look for a configuration that promotes mixing. Passive mixing devices inserted to the oxidizer flow path or changes in the pre-combustion chamber area geometry can be used to produce such disturbances.

A cold flow experiment was designed for investigating the mixing process in a hybrid rocket. Operating a chemically inert experiment is simpler, safer and less costly than firing the real motor. While heat release will influence the flow field in the chemically reacting case, the flow patterns should be qualitatively similar to those in the cold flow situation, and the mixing aspect of the flow field can be captured in the non-reacting environment. A configuration that produces a high degree of mixing in the cold flow case is expected to do so in the presence of chemical reactions.

This document describes the cold flow experiment performed to study mixing in a hybrid motor, and a numerical simulation of the experiment.

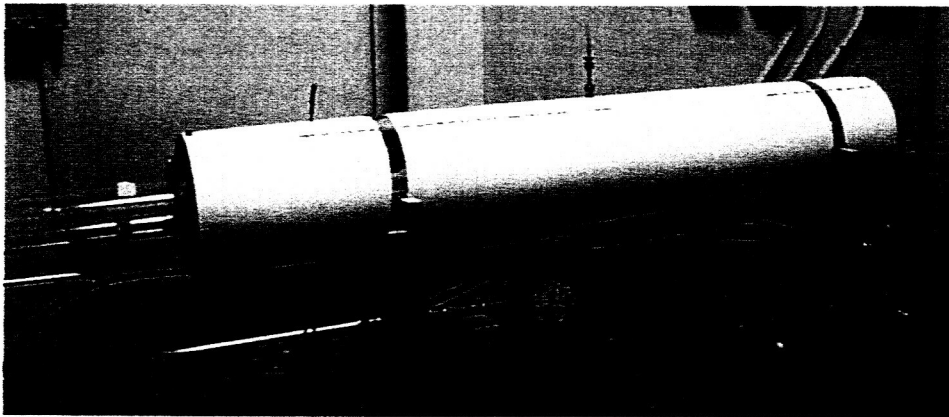


Figure 1: Mixing experiment apparatus

Experiment setup

Figure 1 shows a photograph of the experimental apparatus. Both the fuel and the oxidizer are simulated with air. The fluid representing oxidizer flows axially in a pipe, while the fluid representing fuel is injected from the pipe walls. The radial flow is maintained at a temperature slightly higher than that of the axial flow. Mixing is inferred from temperature profile measurements taken at the pipe exit.

Figure 2 shows a schematic of the apparatus. The axial flow is driven by a high pressure blower, and the flow rate is controlled by adjusting the speed of a motor driving the blower. The fluid first flows through a long, 3 inch diameter inlet pipe. This will produce a fully developed velocity profile at the downstream end of the inlet pipe. The pipe also contains honeycomb and screens for flow conditioning. After the inlet pipe, the flow expands through what would be the pre-combustion chamber area in a hybrid motor, and finally enters the main pipe, which corresponds to the fuel port of a motor. The diameter of the pipe is 4 inches, and it is 44 inches long. The fluid representing the fuel comes

from a compressed air supply. The flow rate is controlled by a choked orifice. Before being injected into the axial flow, the fluid enters an outer chamber surrounding the main pipe through a distribution tube. The tube contains holes that increase in size along its length to distribute the fluid into the chamber as uniformly as possible. The fluid then enters the main pipe through the wall. The pipe wall is a perforated metal cylinder, and it is surrounded by layers of foam and low-permeability cloth. The foam and the cloth create a significant pressure difference between the interior of the pipe and the outer chamber. The pressure drop between the pipe centerline and the outer chamber for a typical run is about 350 Pa, while the difference between the maximum and minimum pressure inside the pipe is of the order 20 Pa. This ensures that the radial flow at the wall is as uniform as possible along the length of the pipe. After entering the pipe, the axial and radial flows can interact and mix, and finally exit the pipe.

The temperature of each air stream is actively controlled. A 1000 W heater located downstream of the choked orifice controls the temperature of the radially injected fluid. The heater is connected in series with a solid state relay. An Omega CN8501 temperature controller unit switches the relay on and off, maintaining the fluid at a desired setpoint. The temperature of the axial stream is set by another controller of the same model. A 500W heater is located upstream of the blower. The axial flow is typically maintained at a degree or two above the ambient temperature, and the radial flow is heated by an additional 10 K.

The geometry of the apparatus replicates that of the Ames Hybrid Combustion Facility, with the exception that it does not include a post-combustion chamber or a nozzle. The ratio of axial to radial mass flow rates in the experiment is $m_{\text{axial}}/m_{\text{radial}}=2.4$, corresponding to a typical oxidizer to fuel ratio in test firings at the Ames facility. The Reynolds number based on the pipe diameter and average velocity of the incoming axial flow is approximately $3 \cdot 10^5$.

The primary measurements have been mean temperature and velocity profiles at the exit of the pipe. Temperature profiles are recorded with an RTD probe, and velocity profiles are measured with a Pitot static probe used in conjunction with a high-precision Combist manometer. To measure the profiles, the appropriate sensor is mounted on a traverse. After each data point is recorded, a stepper motor moves the sensor by a distance set by an indexer. In addition to the movable sensor at the pipe exit, the experiment has four additional, permanently mounted RTD sensors. One of these sensors is mounted on the perforated wall of the pipe, and another one measures the temperature in the outer chamber. Two sensors monitor temperatures of the incoming air streams, and provide process values for the two temperature controllers.

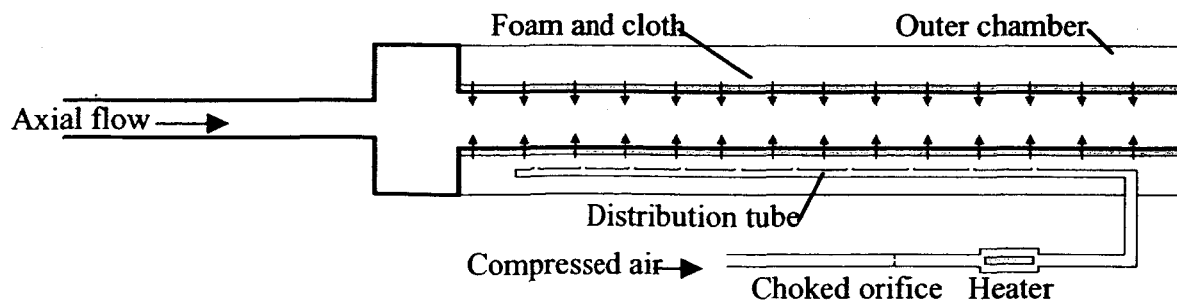


Figure 2: Schematic of experimental apparatus

Computational Model

The numerical simulation was a Reynolds Averaged Navier Stokes solution for the geometry of the experiment. Star-CD, a commercial CFD package was used for pre-processing, solving and post-processing. The turbulence model was the standard $k-\epsilon$ model with wall functions. The flow was assumed to be axisymmetric, and cylindrical coordinates were used in the model. The calculation was performed in a wedge extending 10 degrees in the azimuthal direction. The solution was performed on two different grids. The base grid had approximately 12000 cells. This grid was refined by subdividing each cell into four cells, resulting in a grid with approximately 48000 cells. Views of a section of the base grid and refined grid are shown in figures 3 and 4. The computation required boundary conditions for both temperature and velocity. In the azimuthal direction, a symmetry boundary condition was specified for both variables on planes at ± 5 degrees. A fixed axial velocity profile and a constant temperature were specified at the inlet, the temperature and radial velocity of the injected fluid were fixed at the pipe wall, and the mass flow rate was specified at the exit.

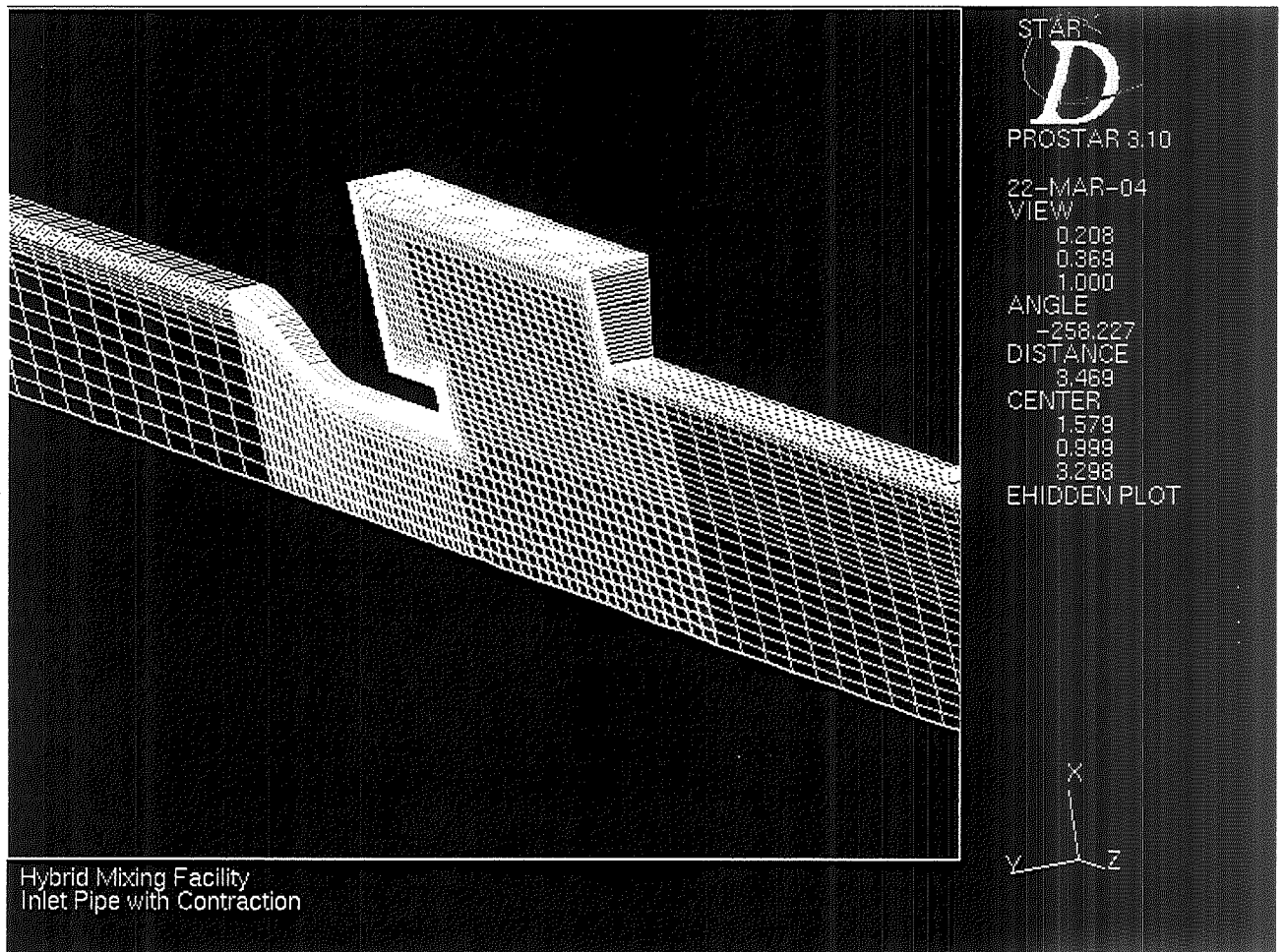


Figure 3: View of a part of the computational mesh, base grid

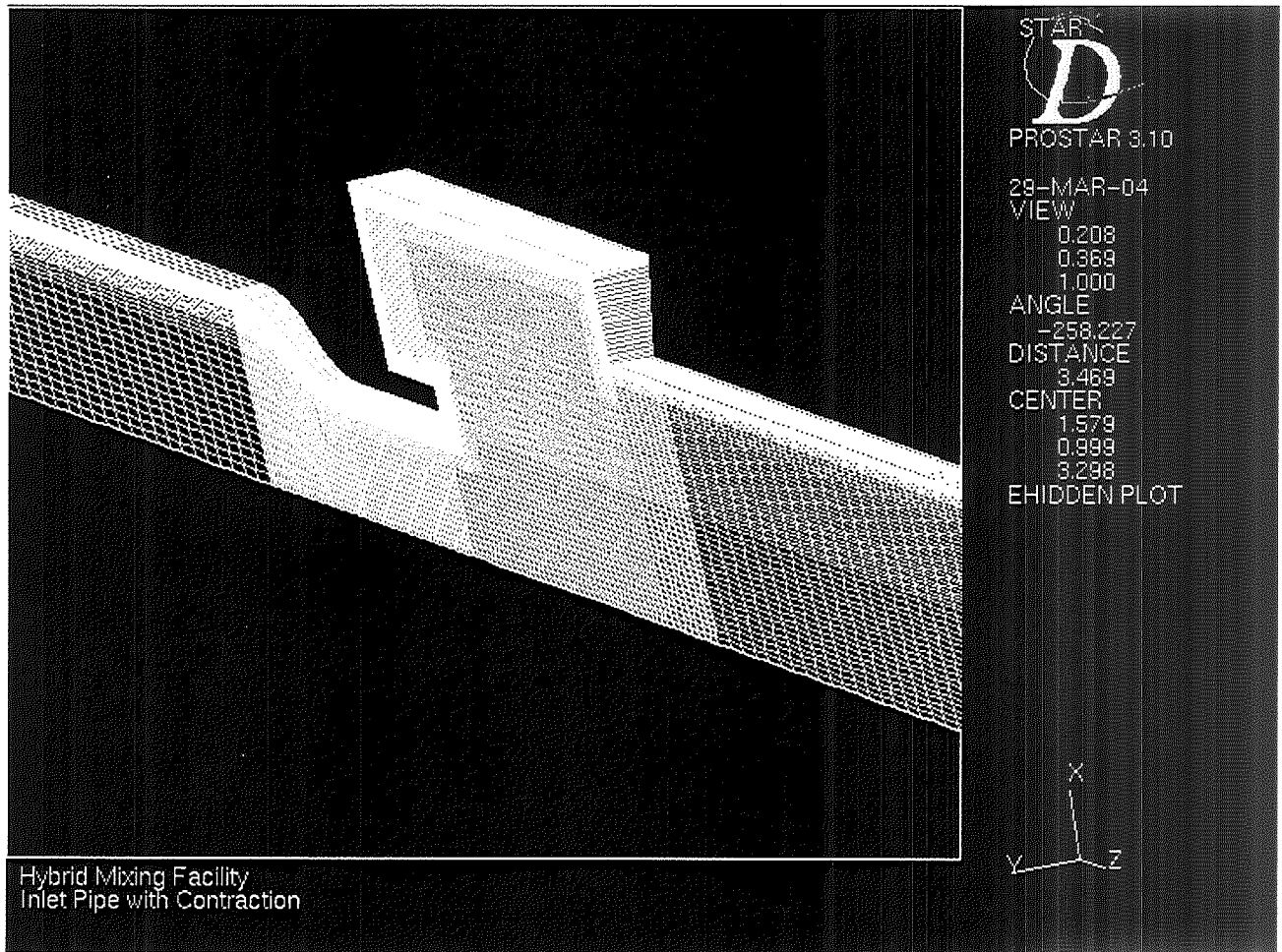


Figure 4: View of a part of the computational mesh, refined grid

Results

Comparison of Experimental and Computational Results

Figure 5 shows a comparison of calculated and experimentally measured velocity and temperature profiles. The axial velocity at the pipe exit is plotted as a function of the radial distance from the pipe centerline. The temperature data is presented in the non-dimensional form, $(T - T_{ref}) / (T_{wall} - T_{ref})$. T_{ref} is the temperature of the incoming axial flow, T_{wall} is the temperature of the perforated pipe wall, and T is the temperature measured at the pipe exit. The data corresponds to the baseline case with undisturbed axial flow. Computational results are shown for both the base grid and the refined grid. The match between the computational and experimental results is currently rather poor. Refining the grid did not bring the computational solution closer to the experimental results. This indicates that further refinement of the grid is not expected to produce a better match

between the two. Compared to the computation, the experimental results show a significantly larger difference between the centerline temperatures and the temperature at region near the wall. It appears that the computation is producing more turbulent diffusion than the experiment. Fluid injected from the wall is more likely to end up near the centerline according to the computation than according to the experimental data.

Figure 6 shows a comparison between the calculated and experimentally measured static pressure as a function of axial distance in the pipe. This indicates that in the calculation, a larger pressure gradient is required to drive the flow through the pipe than what is observed in the experiment. These discrepancies have not been satisfactorily explained. However, only the standard $k-\epsilon$ turbulence model has been attempted, and it is possible that another turbulence model might produce better results. It is also possible that the flow in question, particularly with the complex separation-reattachment zone at the expansion area upstream of the pipe is too complicated to be handled by a steady RANS calculation. More powerful computational tools may be required to improve the match between experiment and computation.

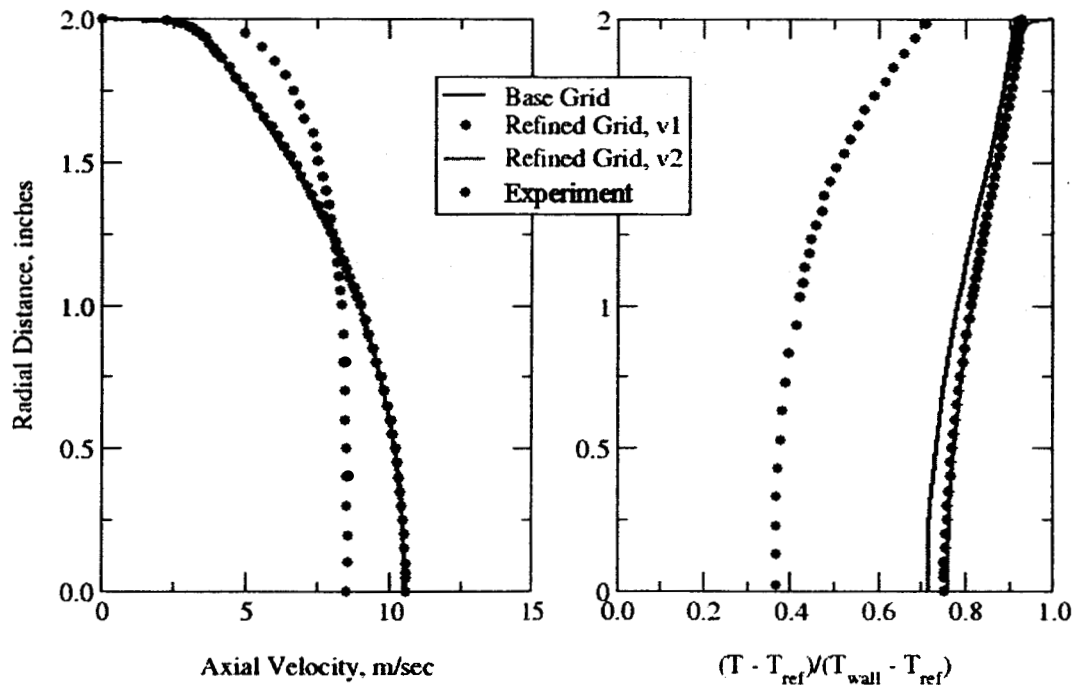


Figure 5: Comparison of experimental and computational results, velocity and temperature profiles at exit plane

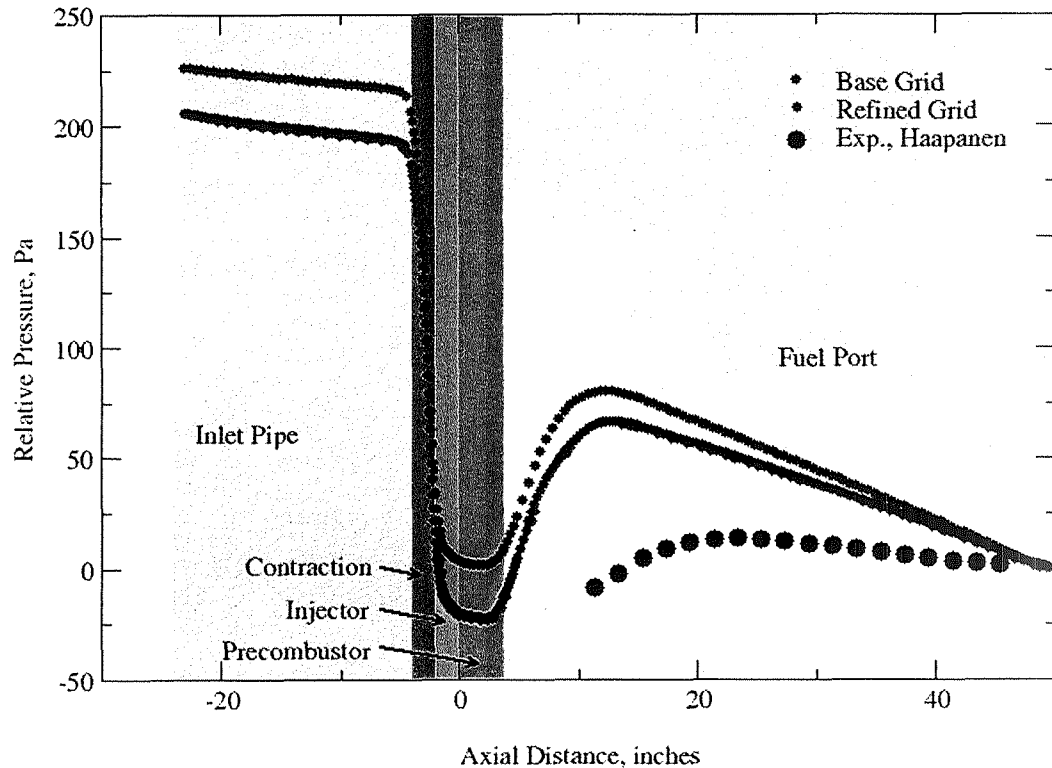


Figure 6: Comparison of experimental and computational results, axial pressure profile

Testing of Flow Disturbance Generators

Figures 7-11 show images of the devices that have been tested. The devices are placed in the path of the axial stream, either at the downstream end of the inlet pipe or just upstream of the main pipe. The intent is to enhance mixing by generating disturbances to the axial flow. None of the devices tested so far have produced a significant deviation to the temperature profile compared to the baseline case, an example of which is demonstrated by the data in figure 12 showing the baseline temperature profile and the profile produced by one of the devices (shown in figure 7).

One important point to be made is that a mean temperature measurement does not indicate whether or not the fluid is mixed at molecular level. The mean temperature of a flow of warm and cool fluid blobs may be the same as that of a flow of homogeneously mixed fluid. The mean temperature profile can only indicate how likely it is for the warm fluid entering through the wall to end up near the centerline, or for the cooler fluid to find its way from the centerline to a region near the wall. A measurement of the fine scale mixing should be incorporated to truly distinguish between mixed and unmixed fluid. The mean temperature profile by itself can only provide information about the bulk movement of the fluid in the radial direction. If the mean temperature between the wall and the centerline is small, the cool fluid has been transported towards the wall, and the warm fluid from the walls has moved towards the centerline. On the other hand, if the mean temperatures at the centerline and near the wall are very different, the cool fluid

had stayed near the centerline, and the warm fluid still resides next to the wall. Ultimately, it might be desirable to achieve both an increase in the bulk transport of fluid in the radial direction as well as a high degree of mixing at the molecular level. However, just increasing the molecular mixing with no change in the mean profile would be a good start. The fact that the devices tested so far have not significant differences in the mean temperature profile does not necessarily mean that they did not produce any changes to the flow field.

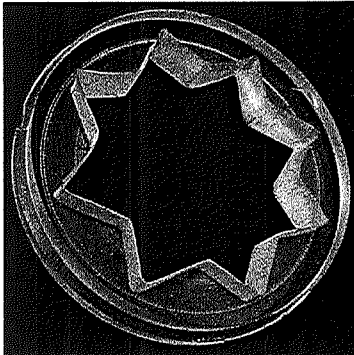


Figure 7

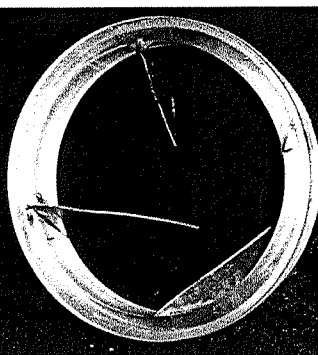


Figure 8

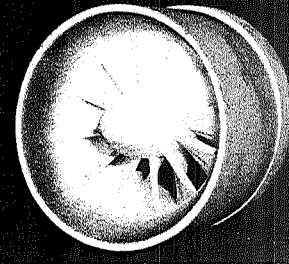


Figure 9

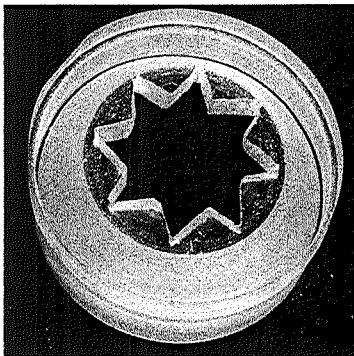


Figure 10

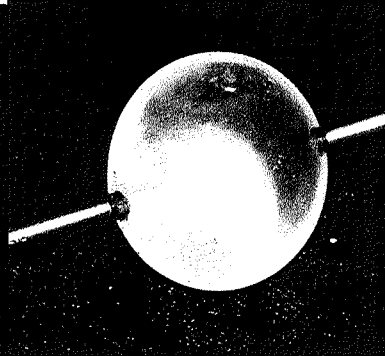


Figure 11

Figures 7-10: Flow disturbance generators. 7: Serrated disturbance generator 1, attaches to inlet of the main pipe, 8: Vortex generator, attaches to inlet of the main pipe, 9: Swirl generator, inserts into inlet pipe near its exit, 10: Serrated disturbance generator 2, inserts into inlet pipe near exit, 11: Spherical disturbance generator, inserts into flow path upstream of main pipe entrance

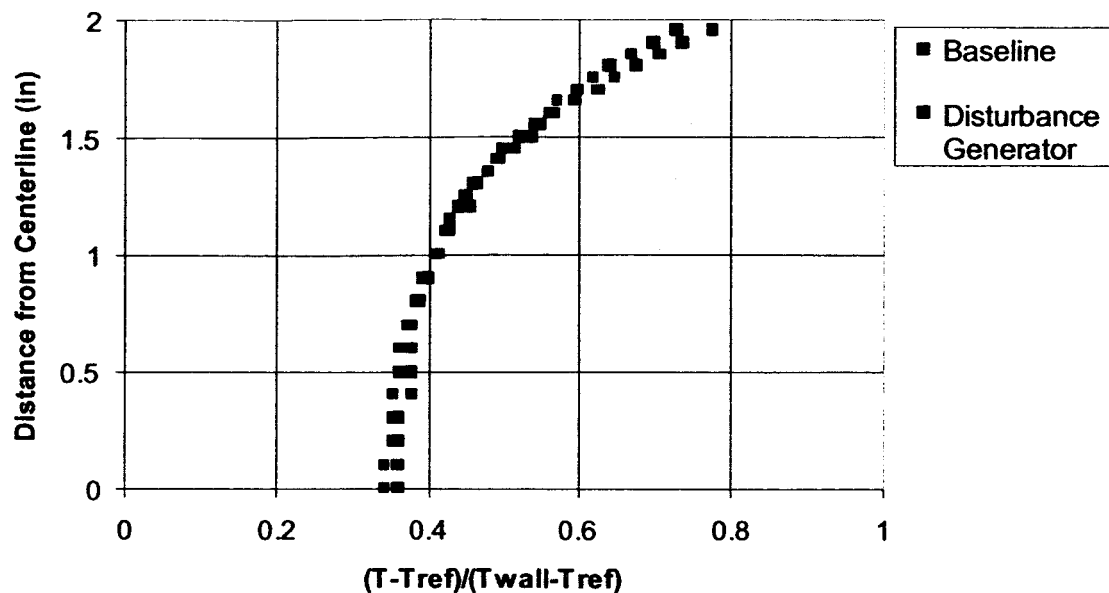


Figure 12: Temperature profile at the exit plane with and without the disturbance generator shown in figure 7

Conclusions and suggestions for further work

Efficiency is an important area of current hybrid combustion research. A cold flow experiment and a related numerical simulation motivated by hybrid rocket efficiency were described in the preceding paragraphs. The experimental and computational results do not match very well, and this issue remains to be resolved. An LES calculation or possibly a different turbulence model may be needed to improve the agreement between the two.

There is a slight ambiguity in relating results from the cold flow experiment to the actual hybrid motor. While enhanced mixing is in general related to an improvement in the efficiency of a chemically reacting system, in the case of hybrid combustion, better mixing may result in a higher combustion temperature, which will increase the regression rate. If the cold flow experiment shows improved mixing for a particular device, it is hard to say if a similar device in the real motor would result in higher efficiency, or if it would instead increase fuel production at the fuel surface, in which case the efficiency could remain unchanged or even decrease. However, despite the ambiguity, it is worthwhile to do testing with the cold flow experiment. If a device does not produce significant changes to the flow field in the cold flow case, it is unlikely to affect the flow field in a real motor either, and it is probably not worth the effort and resources to test a similar concept in the combustor environment. On the other hand, if a device substantially alters the mixing behavior in the cold flow case, it is more likely to cause some type of a change to the flow in the motor. However, further testing in the reacting flow will be necessary to determine the precise effect.

If it turns out that influencing the flow by upstream disturbances can not produce significant enough changes to the flow field, one option is to try to influence the mixing downstream of the port. If this idea is pursued, a section corresponding to a post-combustion chamber could be added to the cold-flow experiment downstream of the main pipe. In this case, there would be a more direct correspondence between the results of a cold flow experiment and the hybrid rocket efficiency. If the flow in the port is unchanged from one case to another, the fuel production at the wall should not be influenced. Enhanced mixing in the cold flow case should correspond to higher efficiency in the reacting case.

Another possibility to be pursued in the future is addition of flow visualization to the experiment. Seeing the flow patterns may help in the design of new disturbance generating devices.

Although the experiment has not yet produced positive results in terms of finding a way to mix the axial flow with the radially injected fluid more effectively, future efforts may turn out more fruitful. None of the devices tested have produced large changes in the mean profiles. However, some measure of molecular mixing should also be obtained for assessing their effectiveness. If future work is pursued with this experiment, it is important that this measurement is added.

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Transient Modeling of Hybrid Rocket Low Frequency Instabilities

M. Arif Karabeyoglu^{*}, Shane De Zilwa[†], Brian Cantwell[‡] and Greg Zilliac[§]

Abstract

A comprehensive dynamic model of a hybrid rocket has been developed in order to understand and predict the transient behavior including instabilities. A linearized version of the transient model predicted the low-frequency chamber pressure oscillations that are commonly observed in hybrids. The source of the instabilities is based on a complex coupling of thermal transients in the solid fuel, wall heat transfer blocking due to fuel regression rate and the transients in the boundary layer that forms on the fuel surface. The oscillation frequencies predicted by the linearized theory are in very good agreement with 43 motor test results obtained from the hybrid propulsion literature. The motor test results used in the comparison cover a very wide spectrum of parameters including: 1) four separate research and development programs, 2) three different oxidizers (LOX, GOX, N₂O), 3) a wide range of motor dimensions (i.e. from 5 inch diameter to 72 inch diameter) and operating conditions and 4) several fuel formulations. A simple universal scaling formula for the frequency of the primary oscillation mode is suggested.

1) Nomenclature:

A :	Pre-exponential coefficient	k :	Blowing parameter exponent
A_p, A_n :	Port and nozzle throat areas	$\hat{j}$:	Laplace transform of oxidizer mass flux perturbation
B :	Blowing parameter	L, L_m :	Length of the fuel port and motor
C :	Specific heat of fuel	L^* :	V_m/A_n
c' :	Boundary layer delay time coefficient	$\dot{m}_o, \dot{m}_t$:	Oxidizer and total mass flow rate
C_f, C_H :	Skin friction coefficient and Stanton number	n :	Mass flux exponent
c_{exp}^*, c_{theo}^* :	Measured and calculated characteristic velocities	O/F :	Oxidizer to fuel ratio
$\bar{D}$:	Average port diameter	P_c :	Average chamber pressure
E_a :	Activation energy	$\dot{Q}_c, \dot{Q}_w$:	Convective and total wall heat fluxes
E_L, E_E :	Energy Parameters	$\dot{r}, R$:	Dimensional and non-dimensional regression rate
f, f_{1-L} :	Primary oscillation frequency, first acoustic mode frequency	Re_z :	Reynolds number
F_T :	Thermal system transfer function	R_g, R_u :	Specific and universal gas constants
F_{TC} :	Thermal-Combustion coupled system transfer function	RT_{ave} :	Average gas constant temperature product
G_o, G_t :	Oxidizer and total mass fluxes	s :	Laplace transform variable
h_v, L_v :	Total and latent heats of gasification	t :	Time
		T :	Temperature
		u, v :	Axial and normal velocities
		U^* :	Diffusion speed

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V_p, V_m :	Port and motor volumes
z :	Axial distance along the port
δ :	Boundary layer thickness
ΔP_{osc} :	Chamber pressure oscillation amplitude
γ :	Ratio of specific heats
κ :	Thermal diffusivity of fuel
ρ_f, ρ :	Fuel and average gas densities
τ_H, τ_d :	Boundary layer response time and thermal lag time
τ_σ :	Shear stress
σ_1, σ_2 :	Gas phase response coefficients
μ :	Average gas viscosity
<u>Subscripts:</u>	
a :	Ambient
1 :	First perturbation variable
e :	Free stream value
ref :	Reference quantities
s :	Surface
<u>Superscripts:</u>	
$-$:	Non-dimensional variable
$\wedge$:	Laplace transform variable

2) Introduction:

The allowable thrust oscillations for an operational propulsion system are limited by certain practical considerations including acceleration loads on the vehicle structures and also on the payload. This makes combustion stability a critical issue during the development of a new propulsion system. The combustion instabilities of solid and liquid systems have been studied in depth and partially successful theories along with certain practical rules for designing stable systems have been developed^{1,2,3}. Even with this extensive knowledge base, development of stable rocket systems requires significant amount of time and resources during the design and testing phases of a new system. Even though the transient operation and instability mechanisms of hybrids have not been explored as extensively as the more mature chemical systems, past experience shows that hybrids do not have the catastrophic instabilities that liquid and solid systems may potentially present.

In fact, hybrid systems typically show finite amplitude (i.e. 2-20% rms of mean chamber pressure) low-frequency chamber pressure oscillations^{4,5,6,7,8}. This most common hybrid instability is in the form of limit cycle oscillations with frequencies much smaller than the first

longitudinal acoustic mode of the combustion chamber.

Even though the exact cause of the low frequency hybrid instabilities is not yet known, a few plausible theories exist^{5,9,10,11,12}. Most of the suggested theories lack the mathematical formalism that exists in the solid and liquid instability models. The fundamental difficulty in producing a mechanism that would generate instabilities comes from the fact that hybrid burn rates are typically independent of the chamber pressure. This, for example, eliminates the pressure coupling between the thermal lags in the solid and the chamber gas dynamics which is the source of the L* instabilities in solid rockets¹.

One particular theory, the TC coupled model^{13,14}, which is based on the coupling of thermal lags in the solid and the lags in the gas phase transport, has predicted (based on a mathematical formalism) fuel mass production oscillations that would drive the observed pressure oscillations. The purpose of this paper is to extend the TC coupled theory to include the effects of the chamber gas dynamics and compare the theory predictions with the motor test data. The merits and shortcomings of the TC coupled model will also be discussed.

3) Transient Phenomena in Hybrid Rocket Motors:

Our approach to modeling the dynamic behavior of hybrids is to isolate the subsystems of the motor and consider every single subsystem individually. For a full description of motor transients one has to consider the following subsystems.

a) **Feed System:** In a hybrid motor, the liquid (or gaseous) oxidizer needs to be fed in to the combustion chamber through a feed system. In reality the feed system response time is finite due to the capacity of the elements in the system. Accurate response depends on the details of the system, which is likely to be significantly different for every design. For this reason we will bypass the feed system dynamics in our investigations by assuming the oxidizer mass flow rate as the input parameter. This is a reasonable assumption for most systems because isolating elements (i.e. cavitating venturi, sonic orifice) are present.

b) **Vaporization of the Liquid Oxidizer:** Complete vaporization of the oxidizer droplets in

the combustion chamber requires a certain characteristic time depending on parameters such as the droplet size and the thermal/flow environment of the space where droplet vaporization takes place. For the purposes of this paper we will assume that the oxidizer is in gaseous phase when it enters the port.

c) Diffusion and Combustion in the Boundary Layer: It requires some time for the hybrid boundary layer properties to adjust to the changes in the port velocity or the fuel blowing from the surface. The associated dynamics has been discussed in detail in Refs. 13, 14 and will be briefly covered in section 5 of this paper.

d) Thermal Response in the Solid Grain: A change in the wall heat flux to the hybrid fuel grain can not be followed immediately by fuel production due to the finite heat capacity of the solid fuel.

e) Chamber Gas Dynamics: The chamber pressure responds to the changes in the mass flow with a time scale proportional to the filling time of the chamber. Acoustic response is also resolved in this sub-system.

In the subsequent sections the last three of these sub-systems (sections c-e) will be modeled and investigated individually. Eventually, these subsystems will be coupled to give the overall system response. The order of magnitude estimates of the time scales of some of the important processes encountered in a typical hybrid motor are listed in Table 1. In our modeling, solid and gaseous kinetic times are assumed to be fast compared to the other relevant time scales.

4) Thermal Lag Model:

The regression rate of the hybrid fuel grain cannot respond to the changes in the surface heat flux instantaneously due to the finite thermal conductivity of the solid fuel. A complete transient model for a hybrid rocket system requires a dynamic model to predict the regression rate history for a given wall heat flux schedule. In order to serve this purpose a thermal lag model that models the nonlinear heat conduction in the fuel grain with a moving boundary has been formulated and reported in Refs.13 and 14. The schematic of this particular model is shown in Figure 1.

In the thermal lag model, the gasification and pyrolysis reactions at the surface are both modeled by an exponential expression of the Arrhenius type. For chemical reactions, the exponential constant is an activation energy, whereas in vaporization it is the latent heat. In order to describe this behavior, we assign an average effective activation energy, E_a , resulting in:

$$\dot{r} = A e^{-E_a/R_s T_s} \quad (1)$$

Here T_s is the surface temperature, $\dot{r}$ is the regression rate and R_s is the universal gas constant. The mathematical formulation and various solution techniques for the thermal lag model are presented in Ref 13. One solution of interest is obtained by perturbing the full nonlinear system around the nominal operating point. The linear initial-boundary value problem defined for the first order perturbation quantities has been solved with use of the Laplace transformation technique. The transfer function between the regression rate perturbation and the applied heat flux perturbation can be written as

$$F_T = \frac{\hat{R}_1(s)}{\hat{Q}_1(s)} = \frac{2E_{E_s}s}{(1+\sqrt{4s+1})(s+E_{E_s})-2E_{E_s}+2E_L E_{E_s}s} \quad (2)$$

Note that $\hat{R}_1(s)$ and $\hat{Q}_1(s)$ are the Laplace transforms of the non-dimensional regression rate perturbation and the wall heat flux perturbation, respectively. Here the following non-dimensional parameters are defined for convenience.

$$E_{E_s} = \frac{E_a \Delta T}{R_s (T_s)_{ref}^2} \quad E_L = \frac{L_v}{CAT}$$

$$\Delta T = (T_s)_{ref} - T_a \quad \bar{Q} = \frac{\dot{Q}_w}{\dot{Q}_{ref}}$$

$$R = \dot{r}/\dot{r}_{ref} \quad \dot{Q}_{ref} = \dot{r}_{ref} \rho_f CAT$$

to investigate the stability character of the TC coupled system. The commonly used method is to map the poles of the transfer function in the s -plane. Particularly, the real component of a certain pole of a transfer function indicates amplification rate associated with that pole. Similarly the imaginary part represents the oscillation frequency.

First a system with zero boundary layer delays (i.e. $\tau_{M1} = \tau_{M2} = 0$) is determined to be always stable. In fact plot of the transfer function for a system with typical parameters shows no poles in the s plane (only a zero at $(0, 0)$) indicating no sign of instability (i.e. See Figure 3). If a positive delay is introduced between the regression rate and wall heat flux, a series of unstable poles is generated. The example case with a 38 msec delay is shown in Figure 4. Note that all the other parameters are kept identical to the case with no delays. Even though there exists an infinite series of poles generated (at the same amplification rate) we will only concentrate on the pole with the lowest frequency (fundamental mode). We believe that the higher order modes are an artifact of using a simple delay instead of the full dynamics for the boundary layer transients. Moreover the higher frequency modes, even if they exist in a real system, are likely to be damped more effectively compared to the fundamental mode.

It is important to identify the necessary conditions that must exist for the generation of TC coupled instabilities. A careful examination of the denominator of the transfer function, Eq. 6, shows that for the instability to exist blowing exponent, k , activation energy, E_a and delay time, τ_{M2} must be nonzero. This indicates that the instability is a result of the coupling of three physical phenomena: blocking of heat transfer by radial injection of fuel mass, thermal transients in the solid and boundary layer dynamics. The gas phase/solid phase coupling mechanism captured by the TC coupled model is shown in Figure 1.

Next we consider the effect of various parameters on the TC coupled instability. It can be shown that the effect of the regression rate mass flux exponent n , heat of vaporization of the fuel, L_v , the heat capacity of the solid fuel, C , and the surface temperature, T_s , on the oscillation frequency and amplitude are negligible for the range of these parameters that are commonly found in hybrid systems. Also TC coupled system characteristics are completely independent of the

solid density, ρ_f and the heat diffusivity in the solid, κ .

The effect of the activation energy on the amplification and frequency is shown in Figure 5 for $\tau_{M2} = 38$ milliseconds and $k = 0.68$. As indicated by the figure, for activation energies commonly observed in hybrids, ranging from 5 kcal/mole to 60 kcal/mole, the variation in the oscillation frequency is relatively small. The amplification increases with increasing activation energy. For this specific case the systems with activation energies larger than 3 kcal/mole show positive amplification and unstable behavior. Decreased stability at higher activation energies is expected since at high E_a values the surface temperature and regression rate are more closely coupled. This coupling is one of the necessary ingredients for the generation of the instabilities. In fact in the extreme case of $E_a = 0$, for which the surface temperature is completely independent of the regression rate, oscillation frequency goes to infinity and the amplification goes to negative infinity.

The activation energies for the polymeric systems that are typically used in hybrids ranges from 10 kcal/mole to 60 kcal/mole. For the nonpolymeric paraffin-based fuel system the activation energy is equivalent to the latent heat of vaporization¹⁸. This value for the paraffin-based fuel formulations is estimated to be 17 kcal/mole (Ref. 13).

Similar results can be obtained for the effect of the blocking exponent, as shown in Figure 6 for $\tau_{M2} = 0.38$ and $E_a = 15$ kcal/mole. For the range of values reported in the literature for k (i.e. 0.68-0.77)^{15,19}, the effect of blowing exponent on the frequency is negligible. The amplification increases with increasing k and systems with k values 0.45 and larger showed unstable behavior. Note that for the physically unrealizable case of $k = 0$ the instability disappears, since one of the necessary coupling mechanisms is eliminated. We finally like to note that even though the blocking effect in paraffin based systems is somewhat reduced due to the two phase character of the flow field, its is still significant enough to establish the coupling between the gas phase and solid phase.

The most influential parameter on the TC coupled frequency is determined to be the boundary delay time, τ_{M2} . Figure 7 shows the

predicted frequency as function of the delay time for three activation energy values, $E_a = 5, 15$ and 50 kcal/mole. As shown in the figure, the effect of the activation energy on the frequency, especially at longer delay times, is small. For the purposes of this paper the effects of activation energy and blowing exponent will be ignored and the frequency will be represented as a function of the boundary delay time alone. The following curve fit equation is suggested to quantify the inverse relationship between the frequency and the boundary layer delay time.

$$f = \frac{0.48}{\tau_{M2}} \quad (7)$$

As shown in Figure 7, this expression represents the predicted frequency to a very high degree of accuracy and will be used for all the hybrid systems that will be discussed later in this paper.

7) Gas Dynamic Model:

In the previous section the dynamic models for the thermal lags and the hybrid combustion are summarized. In addition, these subsystem models are coupled to obtain the response of the fuel mass generation to the changes in the mass flux in the port. However, practically significant parameters of the rocket operation such as the chamber pressure, specific impulse and thrust cannot be obtained solely from the TC coupled system. These variables can only be determined after introducing a model for the gas dynamics which will be discussed in this section. A gas dynamic model, in general, should use the oxidizer mass flow rate and the fuel mass generation rate as the inputs and it should yield the parameters that are more closely related to the performance of the motor such as the chamber pressure and the motor O/F ratio as the outputs.

In our specific model (2V-port model) we divide the chamber into three components, pre-combustion chamber, port volume and the post-combustion chamber (i.e. see Figure 8). We treat the pre and post combustion chambers as zero dimensional volume elements, but consider the variation of state variables along the axis of the port. The mathematical formulation is obtained after the application of the conservation laws for the three volume elements. The set of equations resembling our model is solved analytically after linearization and as a result of that operation, a

transfer function for the gas dynamic subsystem is obtained. Some numerical simulations are also performed to determine the validity of the approximations used in the perturbation solutions. Note that a detailed description of the gas dynamic model is given in the Appendix of this paper.

The following conclusions can be drawn from the results of the gas dynamic model as given in detail in the Appendix of this paper and in more detail in Ref. 13.

- Gas dynamic model captures the filling/emptying dynamics and also the longitudinal acoustic response of the hybrid combustion chamber. As can be seen from Figure 13 acoustic modes are stable (well damped). No unstable oscillation modes are generated by the gas dynamic system by itself.
- The nonlinear disturbances excite the chamber acoustic modes. Simulations showed that a sudden pulse in the oxidizer mass flow rate excites the acoustics modes. These modes decay in a relatively short period. This ringing phenomenon shows that the longitudinal acoustic modes can be driven by nonlinear low frequency oscillations. Namely the nonlinear low frequency pressure waveforms continuously excite the acoustic modes of the chamber. AMROC motor DM-01 showed at least 5 longitudinal acoustics modes. All those were well damped. The gas dynamic model when corrected for isentropic speed of sound predicts the acoustics frequencies of the DM-01 motor to a high level of accuracy (See Table 7).
- Gas dynamic model coupled with the boundary layer dynamics ($\tau_{M1} \neq 0$) does not produce any instabilities (no extra poles are generated in the s plane). This conclusion is based on transfer function analysis and also numerical simulations. This important result highlights the importance of the thermal lag system (i.e. and its unique dynamic capability of producing a phase lead) in producing a coupling mechanism to generate low frequency unstable modes.

8) Thermal-Combustion-Gas dynamic (TCG) Coupled System:

In this section we integrate the gas dynamic system to the TC coupled system to establish the TCG coupled response (i.e. See Figure 2). The TCG coupled system yields the performance of the rocket (such as chamber

pressure) for some given input of oxidizer mass flow rate, which is a fundamental control parameter in hybrid rockets. It is fair to state that the TCG coupled system represents the most fundamental dynamic behavior of a hybrid rocket.

The schematic of the hybrid subsystems and the information flow between the subsystems in the context of TCG coupling are shown in Figure 2. Note that the lower block in the schematic represents the TC coupled system. The input for the TC coupled system is the local mass flux information and the output is the mass generation or the regression rate. The upper block shows the 2V-port gas dynamic model. The gas dynamic model takes the oxidizer flow rate and the mass generation rate information and reveals the important performance parameters such as the chamber pressure and specific impulse. The TCG coupled system can be considered as an overall model for the dynamics of a hybrid propulsion system that utilizes gaseous oxidizer which is delivered by an isolated feed system.

The generalization of the TCG coupled system to a liquid hybrid with significant feed system dynamics can be achieved easily. Namely two more modules for the liquid droplet evaporation and the feed system dynamics must be added to the front end of the TCG model. We believe that the fundamental phenomenon generating the low frequency instabilities of hybrid systems does not involve either of the evaporation lags or the feed system dynamics.

In the TCG coupled system the gas dynamic module converts the fuel mass oscillations produced by the TC coupled system into the chamber pressure oscillations. It has been determined that in the process of conversion, the oscillations produced by the TC coupled system (i.e. frequencies and amplification rates) are not altered. This fact is demonstrated in Figure 9, which shows a plot of the TCG system transfer function for the TC coupled system depicted in Figure 4. The same plot also indicates poles associated with the filling/emptying mode (first order non-oscillatory mode) and the longitudinal acoustic modes with negative real components (damped). TCG coupled system captures all the necessary underlying dynamics (linearized version) of a gaseous hybrid with a decoupled feed system.

9) NASA Ames Paraffin-Based Motor Tests:

A promising fast burning paraffin-based fuel has been tested extensively in the Hybrid Combustion Facility (HCF) at NASA Ames²⁸. The motor outside diameter is 10 inches and gaseous oxygen is the oxidizer for all the tests conducted at the HCF. The pressure time trace of a typical paraffin-based hybrid motor test is shown in Figure 10, whereas the Fourier transform for the same test is given in Figure 11. Note that the chamber pressure traces for classical hybrids as presented in the literature are very similar in nature. As indicated in Figure 10 for a particular case, the feed system and the chamber were completely decoupled for all paraffin-based motor tests used in this paper.

The Fourier transform shows three broad peaks corresponding to the low frequency hybrid instability, the Helmholtz mode and the first longitudinal mode. It has been generally observed that the dominant mode, low frequency oscillations are accompanied by lower amplitude higher frequency acoustic modes. See Ref. 6 for an extensive discussion of the stability characteristics of the paraffin-based motors.

10) Comparison of Theory Results with the Motor Test Data

In this section we will compare the TC coupled theory oscillation frequency predictions with the chamber pressure oscillation frequencies from hybrid motor test data. In order to produce a conclusive comparison, we have selected a large number of tests from several hybrid rocket development programs. The key features of each program is discussed briefly in the following paragraphs.

- AMROC Motor Tests^{4,13}: In this case the reported results are for 4 different thrust class hybrid motors which all utilized LOX/GOX as the oxidizer and HTPB as the fuel. The data from the AMROC tests is given in Table 2.
- HPDP 11 Inch Motor Tests^{5,8}: The oxidizer used in these tests is LOX and GOX and the fuel is HTPB/Escorez formulation. The motor case diameter was 11 inches and the data from 5 tests used in this study is given in Table 3. Only the tests that exhibit combustion instability and performed with single port motors are considered. The runs with the rearward-facing steps at the port entrance are also excluded in this study, since in these tests

the flow field in the port is significantly altered which may cause a major change in the boundary layer delay time coefficient. We also include test #1 of 11 inch LOX motor data in the analysis. We only consider test 1 since it is the only run with a chamber configuration identical to GOX motors used in the calculations.

- JIRAD Motor Tests^{13,21}: The oxidizer used in these tests is GOX and the fuel is HTPB/Escorez formulation. The motor case diameter was 11 inches and the data from 8 tests used in this study is given in Table 4. Only the tests that exhibit combustion instability and performed with single port motors are considered here.
- Arizona State University²²: One test is included in the comparison for a 5 inch diameter nitrous oxide motor. The fuel was HTPB (i.e. See Table 5).
- Paraffin-Based Motor Tests²⁹: These tests are conducted at NASA Ames in a 10 inch diameter test facility. The oxidizer was GOX and the fuel is two separate nonpolymeric paraffin-based formulations (SP-1a and SP-4). The data from 25 motor tests are reported in Table 6.

The prediction of the frequency requires the estimation of the boundary layer delay time for each test from the reported data. Since the reported data is somewhat different for each program different scaling laws for the delay time must be derived.

The first one of those is for the various size AMROC motors which operated at different L^* levels. Since the information on the AMROC motors that can be found in the literature is limited to L^* , c^* and motor O/F ratio, it is desirable to express the boundary layer delay time, Equation 4 (with $z = L/2$) in terms of those variables. The average velocity in the port can be approximated as

$$u_{ave} = \frac{G_o \left[(1 + 2O/F) \right] (1 + O/F) RT_{ave}}{2P_c} \quad (8)$$

where RT_{ave} is an average value in the port. Substituting this expression in the delay formula and using the relations for the total mass flow rate $\dot{m}_t = G_t A_p$ and port volume $V_p = LA_p$ yields

$$\tau_{M2} = c^* \frac{V_p P_c}{\dot{m}_t \left[(1 + 2O/F) \right] (1 + O/F) RT_{ave}} \quad (9)$$

With the use of the total mass flow relation, $\dot{m}_t = P_c A_p / c_{exp}^*$ and the definition $L^* = V_m / A_p$, the delay equation can be written as

$$\tau_{M2} = c^* \frac{V_p}{V_m} \frac{\left[(1 + O/F) \right] (1 + 2O/F)}{RT_{ave}} L^* c_{exp}^* \quad (10)$$

Here V_p/V_m is the ratio of the port volume to the motor volume, which is estimated to be approximately 0.8 for AMROC DM-01 motor. We assume that all AMROC motors possess the same average gas constant temperature product, RT_{ave} and volume ratio, V_p/V_m . After noting that all motors operate at very similar O/F ratios and c_{exp}^* values, it can be stated that in the series of tests the boundary layer characteristic delay time is proportional to L^* .

In order to reduce data form the rest of the tests, a similar relation for the boundary layer delay time in terms of the chamber pressure and port flux levels must be derived. The port velocity can be replaced by an average value $(G_o + G_t)/2\rho_{ave}$ with ρ_{ave} given by the gas law $P_c = \rho_{ave} RT_{ave}$ to yield finally for the boundary layer lag:

$$\tau_{M2} = c^* \frac{LP_c}{(G_o + G_t) RT_{ave}} \quad (11)$$

Equation 11 gives the scaling law for the boundary layer delay in terms of the operating points of the rocket motor and also with the size of the motor. This states that delay increases with increasing chamber pressure, port length and decreases with increasing mass flux in the port. An important conclusion that can be drawn from those observations is that an increase in the chamber pressure or a decrease in the port mass flux act to decrease the oscillation frequency of this type of TC coupled system.

The total mass flux can be eliminated from Equation 11 by introducing the average oxidizer to fuel ratio of the motor.

$$\tau_{M2} = c^* \frac{LP_c}{\left(2 + \frac{1}{O/F} \right) G_o RT_{ave}} \quad (12)$$

Equation 12 will be used to reduce the paraffin-based motor data.

Note that equations 10, 11 and 12 are equivalent and the constant multiplier c' is identical for all of these expressions.

The average temperature gas constant product that appears in the denominator of all the boundary layer delay time equations is assumed to be constant for all of the LOX/GOX motor tests that is considered in this paper. This is a fairly good assumption since RT is a weak function of the motor O/F for the practical operating conditions¹³. In fact the maximum expected variation on the RT values for all the LOX/GOX tests used for comparison is predicted to be less than 5%. The absolute value of RT_{ave} is selected to be $6.39 \cdot 10^5 \text{ (m/sec)}^2$ for LOX/GOX motor tests and since the same value is used for all calculations any error in RT_{ave} will only effect the numerical value of the empirical delay constant c' .

In order to double check the RT_{ave} value used in the calculations one could use the first acoustic frequency measured from the motor test data and use that to estimate the speed of sound and finally RT_{ave} based on the following equation.

$$f_{1-L} = \frac{\sqrt{\gamma RT_{ave}}}{2L_m} \quad (13)$$

$$f = 0.48 \frac{RT_{ave}}{c' (V_p/V_m) [(1+O/F)(1+2O/F)] L^* c_{cp}^*} \quad (14a)$$

$$f = 0.48 \frac{(G_o + G_t) RT_{ave}}{c' LP_c} = 0.48 \left(2 + \frac{1}{O/F} \right) \frac{G_o RT_{ave}}{c' LP_c} \quad (14b)$$

The oscillation frequency as a function of the boundary delay time predicted by the TC coupled theory along with data from the hybrid motor development programs are plotted in Figure 12. Note that we have used a boundary layer delay constant value, c' , of 2.01 for all motor tests in order to obtain the best fit between the theory and test results. Please note that this experimental value for c' is the same order of magnitude with the rough estimate (i.e. ~0.55) for a boundary layer with no blowing and no combustion.

Considering the wide range of test conditions, the agreement between the measured pressure oscillation frequencies and the TC

Here γ is the ratio of the specific heats averaged over the length of the motor, f_{1-L} is the first longitudinal acoustic mode and L_m is the effective length of the motor. For a more accurate calculation the complex gas dynamic model discussed in the Appendix of this paper can be used. All of the NASA Ames motor tests with 45 inch long grains had their first acoustic mode at around 370 Hz. For these tests with use of $\gamma=1.25$ and $L_m=1.27 \text{ m}$, one obtains RT_{ave} of $7.1 \cdot 10^5 \text{ (m/sec)}^2$ which is only 10% higher than the assumed value. RT_{ave} for the N_2O test is selected to be $5.1 \cdot 10^5 \text{ (m/sec)}^2$. This is 20% lower than the RT_{ave} for the GOX/LOX system due to the low temperature of the N_2O /hydrocarbon combustion products.

Now Equation 7 can be coupled with the expressions for the boundary delays time to evaluate the frequencies predicted by the TC coupled model.

coupled model predictions is surprisingly good. Note that the comparison includes 43 motor tests

- Using results from 4 different development and research programs
- Using 3 separate oxidizers (i.e. LOX, GOX and N_2O)
- Covering a wide range of motor dimensions (i.e. from 5" OD Arizona state motor up to 72" OD AMROC motor)
- Using many different fuel compositions (i.e. HTPB, HTPB/ESCOREZ, HTPB/DDI/ESCOREZ, 2 paraffin-based formulations)

- Covering a wide range of operating conditions.

Equation 14a indicates that for motors operating at the similar O/F, the oscillations frequency is inversely proportional to the L^* of the motor. This conclusion is consistent with the previous results reported in the literature. Similarly Equation 14b shows for motor operating at equal oxidizer mass fluxes, chamber pressures and O/F the oscillation frequency decreases with increasing fuel grain length. In order to prove that motor length plays a critical role setting the oscillation frequency, we have plotted the Ames motor data, which contains two sets of grain lengths, by removing the length effect from the scaling law. This resulted in an increase in the scatter in the frequency data indicating the importance of the grain length. A similar check to confirm the importance of length can be conducted by comparing the JIRAD and HPDP motor tests (i.e. ran with grains 102 inches long) with the Ames paraffin tests (i.e. ran with grains 32 or 45 inches long) that were conducted under very similar operating conditions. For example comparing test 8 of the HPDP program to paraffin based motor test 4P-01 shows that grain length must be included in the scaling law in order to explain the discrepancy in the oscillation frequencies for these tests. The same conclusion can be drawn by comparing test 2 of HPDP program to the test Ames test 4L-08.

TC coupled theory predicts oscillations of the regression rate/fuel mass generation rate and would be observed directly in a hybrid motor as the radial oscillations of the diffusion flame within the boundary layer. The gas dynamic model converts the fuel mass generation rate oscillations to the chamber pressure oscillations. It is expected that a system operating at higher O/F ratios would produce less chamber pressure oscillations since fuel mass constitute a smaller fraction of the total mass expelled from the nozzle. This fact is observed in NASA Ames motor tests, namely high O/F motors, in general, presented lower amplitudes chamber pressure oscillations compared to the mean chamber pressure.

The simple linear model presented in this paper successfully explains the generation of small amplitude low frequency oscillations and the scaling of the frequency. But it also falsely predicts an indefinite growth of chamber pressure oscillations. In reality, the nonlinear mechanisms that exist in a hybrid motor would limit the indefinite growth of the oscillations and result in

limit cycle oscillations that are commonly observed in motor tests. Moreover the simple theory does not explain when these low frequency instabilities will take place and what their amplitudes would be. It has been shown in the hybrid development programs that the fore end configuration/volume of the motor plays a paramount importance in setting the amplitude of the low frequency instabilities. The motor tests indicated that, at least for gaseous motors, axial injectors result in more stable operation compared to the radial or conical injection of the oxidizer.

A natural explanation for these observations is that the TC coupled instability mechanism almost always exist in hybrid systems, but the amplitude of the limit cycle oscillations will depend on the excitation level by the disturbances in the right frequency range. We believe that the fore end fluid dynamics (i.e. vortex shedding) is the primary source of disturbances in a hybrid rocket. For example changing the injection scheme of the oxidizer from axial to radial, may introduce a flow disturbance component that is preferred by the TC coupled system. If one assumes that all the low frequency oscillations are developed by the fluid mechanics at the fore end of the motor, one would fail to explain the length effect on the oscillation frequency. Also all oxidizer injection schemes under identical motor operating conditions seem to produce the same oscillation frequency. This fact is also difficult to explain by the assumptions that all the oscillations are produced from the fluid dynamic events at the fore end of the motor, since the injection scheme should alter the fluid mechanics significantly (i.e. vortex shedding frequency).

We would like to note that most of musical instruments also work on the same principle²³. For example in a flue organ pipe the column of air inside the pipe is set into vibration (i.e. at its natural frequencies) by an edge tone which is produced by a jet of air that impinges on a carefully designed lip. The vortex shedding at the lip sets the edge tone which itself excites the natural modes in the pipe. Note that in the case of an organ pipe the jet speed and the lip geometry is carefully selected for the pipe length such that the edge tone produced at the lip contains a large component at the natural frequency of the pipe that needs to be excited. In this organ pipe analog, the lip is the fore end of the motor (i.e. geometry, injection scheme, velocity etc) and the organ pipe is the hybrid system modeled by the TC coupled theory.

11) Conclusions:

The following conclusions can be drawn from the results of the TCG coupled model:

- A plausible mechanism that generates low-frequency chamber pressure oscillations is suggested. This mechanism is due to the coupling of the thermal lags, the gas phase combustion and gas dynamic sub-systems of the hybrid rocket. The physical transient model of the motor is presented in a mathematical formalism that allows one to estimate the expected oscillation frequency. The amplitude of the oscillations can not be determined by this linear model.
- The oscillation frequency estimated by the model is in very good agreement with the motor test data from several programs utilizing three distinct oxidizers, several fuel formulations, a wide range of motor dimensions and operating conditions. We suggest the following equation as a universal scaling law for the primary hybrid oscillation frequency:

$$f = 0.239 \left(2 + \frac{1}{O/F} \right) \frac{G_o RT_{arc}}{L P_c} \quad (15)$$

$RT_{arc} = 6.39 \cdot 10^5 \text{ (m/sec)}^2$ for GOX/LOX systems

$RT_{arc} = 5.11 \cdot 10^5 \text{ (m/sec)}^2$ for low energy oxidizers systems such as N_2O

- This mode of instability is common to all kind of hybrids: liquefying or conventional fuels, liquid oxidizer or gaseous oxidizers. It is reasonable to believe that this low frequency mode is present in every hybrid system to some extent. Some motors are more unstable compared to the others because these motors do possess more disturbances to excite the TC coupled oscillations. For example the oxidizer injector configuration or pre-combustion chamber geometry affects the scale and the frequency of the disturbances that would excite the commonly observed low frequency mode. For this reason the design of the fore end of the motor has been critical in controlling the stability of the system.
- Even though the linear theory very successfully predicts the oscillation frequency of the chamber pressure, it fails to establish an estimation process for the amplitude of the oscillations. The theory predicts an unlimited

growth of the oscillations. In reality the amplitude of the chamber pressure fluctuations will be limited by nonlinear effects that are not covered in the linearized TC coupled theory.

- It has been shown that the longitudinal acoustic oscillations that are commonly coexist with the low frequency oscillations can be generated by the excitation induced by the nonlinear waveforms of the low frequency chamber disturbances. In short, it is plausible that high frequency acoustic modes are driven by the low frequency oscillations.

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14) Appendix

In our gas dynamic model the motor is divided into four subsections: 1) pre-combustion chamber, 2) port volume, 3) post-combustion chamber, 4) nozzle. Each component is approximated appropriately with one or zero spatial dimensional models. The subscripts 1 and 2 are used for the physical properties in the pre and post combustion chambers respectively. Various stations of significance are also distinguished by use of subscript notation. Namely the entrance station of the gaseous oxidizer into the system is represented by a subscript o, the nozzle entrance is denoted by n and the port entrance and exit are shown by a and b, respectively. Our models for these subsections are discussed in the next section.

Pre-Combustion Chamber

We use a zero dimensional gas dynamic model for the pre-combustion chamber. Thus the momentum equation and the energy equations reduce to the mean state information of the pressure and temperature in this volume. But, the mass balance equation is nontrivial and it can be written as

$$V_1 \frac{d\rho_1}{dt} = \dot{m}_o - \dot{m}_a \quad (A.1)$$

Note that the size of the volume is assumed to be constant over the burn time. We further assume a constant molar mass and a polytropic process, $P \propto \rho^{\eta_p}$ in the chamber. Here η_p is the polytropic exponent. We believe that these are reasonable assumptions for this relatively cold volume which accommodates negligible combustion. Under these stated conditions and with the use of the ideal gas law, equation A.1 can be reduced to

$$a_1 \frac{dP_1}{dt} = \dot{m}_o - \dot{m}_a \quad \text{where} \quad a_1 = \frac{V_1}{\eta_p R_1 T_1} \quad (A.2)$$

Port Chamber

In our model we treat the port as a quasi-1D tube with continuous mass and heat addition. The conservation of mass can be written as the following differential equation.

$$\frac{\partial \rho}{\partial t} = -\frac{\partial G}{\partial z} + \dot{m}_f \quad (\text{A.3})$$

Here $\dot{m}_f$ is the mass generated at the axial location z of the port. At this stage we prefer to express the mass generation term in the general functional form.

$$\dot{m}_f = f(G, z)$$

The momentum equation in the port can be written in terms of the local mass flux, G , the local pressure, P and the local density, ρ .

$$\frac{\partial G}{\partial t} = -\frac{\partial (G^2/\rho)}{\partial z} - \frac{\partial P}{\partial z} - \left(\frac{C_f C}{2A_p} \right) \frac{G^2}{\rho} \quad (\text{A.4})$$

Note that the last term in this expression is included to capture the effect of the skin friction on the force balance. Here C_f stands for the skin friction coefficient at location z , A_p as the port area and the C for the circumference of the port. All the geometrical properties are assumed to be independent of time and axial dimension. This is a reasonable approximation for typical hybrids which possess a uniform port diameter distribution along the length and also a slow variation of the port geometry¹⁵.

We further assume that the ideal gas law holds locally in the port.

$$P = \rho RT \quad (\text{A.5})$$

The closure of the system requires the energy equation. For the sake of simplicity we replace the energy equation with the following linear variation of temperature gas constant product in the port.

$$RT(z) = (RT)_h + ((RT)_2 - (RT)_h)z/L \quad (\text{A.6})$$

Note that for the simplicity of notation we treat the gas constant temperature product as a single dependent variable.

Furthermore we assume that the gas constant temperature product at the exit of the port volume is a fraction of the nozzle entrance value RT_c which can be calculated from the equilibrium chemistry at the operating O/F ratio of the motor.

$$RT_2 = f_b RT_c \quad (\text{A.7})$$

In short, we specify the whole temperature field in the motor chamber for a given operating point and we ignore the variations in the temperature field during the transients. In that respect 2V-port is an isothermal gas dynamic model.

Post-Combustion Chamber

Similar to the pre-combustion chamber treatment, we assign uniform thermodynamic properties for the gas in the post-combustion chamber volume. Therefore the dynamics can be represented with the mass balance which takes the following form after the fixed volume assumption.

$$V_2 \frac{d\rho_2}{dt} = \dot{m}_b - \dot{m}_n \quad (\text{A.8})$$

Here $\dot{m}_b$ is the mass flow rate entering the hot volume and $\dot{m}_n$ is the mass flow rate exiting the volume through the nozzle. Note that we have inherently assumed that post-combustion chamber behaves like a well stirred reactor with the reactions fast compared to the transient rates of concern. Equation A.8 can be written as

$$a_2 \frac{dP_2}{dt} = \dot{m}_b - \dot{m}_n \quad \text{where} \quad a_2 = \frac{V_2}{RT_2} \quad (\text{A.9})$$

We have approximated the gas constant temperature product in the post-combustion chamber with product at the port exit.

Nozzle

We first assume that the nozzle flow is quasi-steady which is only valid at very slow transients. We also assume that through out the operation of the motor, the nozzle is always choked, namely at any time the ratio of the chamber pressure to the ambient pressure is larger

than the critical value associated with the properties of the expending gas. After these simplifying assumptions the nozzle flow rate can be expressed with use of the classical quasi-1D gas dynamic formula

$$\dot{m}_n = \frac{A_n \Gamma_c P_2}{\sqrt{RT_c}} \quad (\text{A10})$$

where

$$\Gamma_c = \sqrt{\gamma_c} \left(\frac{2}{\gamma_c + 1} \right)^{\frac{\gamma_c + 1}{2(\gamma_c - 1)}}$$

Here γ_c is the ratio of the specific heats of the combustion products.

At moderate frequency oscillations (such as the acoustic modes of the motor), the acoustic admittance of the nozzle must be considered and a more sophisticated behavior must be modeled. For our purposes, it is adequate to adapt Crocco's expression²⁴ for the nozzle transfer function which is valid for small deviations from the mean chamber pressure and for moderate frequencies.

$$\frac{\tilde{\dot{m}}_n}{\tilde{P}_2} = \frac{A_n \Gamma_c \tilde{P}_2}{\sqrt{RT_c}} + k_n \frac{d}{dt} \quad (\text{A11})$$

Here k_n represents the effect of inertia of the gas in the convergent portion of the nozzle in causing a phase difference between the oscillation of the velocity and pressure at the entrance of the nozzle.

$$k_n = -\frac{c_*}{\gamma_c (c_*^2 - u_c^2)} \int \left(\frac{c_*}{u} - \frac{\gamma_c - 1}{2} \frac{u}{c_*} \right) dz \quad (\text{A12})$$

This integration is performed on the whole convergent portion of the nozzle. Note that c_* is the critical sound velocity, u is the local gas velocity in the nozzle, u_c is the gas velocity at the nozzle entrance.

Equations A.9 and A.11 and the ideal gas law can be combined to relate the pressure perturbation directly to the mass flow rate perturbation at the exit of the port.

$$(a_2 + k_n) \frac{d\tilde{P}_2}{dt} = - \left[\left(\frac{V_2}{(RT_c)^2} \right) \frac{d(RT_c)}{dt} + \frac{A_n \Gamma_c}{\sqrt{RT_c}} \right] \tilde{P}_2 + \tilde{\dot{m}}_b \quad (\text{A13})$$

Note that a fraction of the nozzle volume is added to the post-combustion chamber volume to account for the inertial effects of the gas in the nozzle. For typical hybrids the effective nozzle volume is small compared to the post-combustion chamber volume. We finally indicate that for our isothermal treatment the temperature derivative term is identically zero.

Equation A.13 includes some properties of the combustion products which are not known offhand. These properties are calculated from chemical equilibrium considerations with use of the code STANJAN for the selected propellants. A sample calculation result of the product, RT_c with the motor O/F ratio for HTPB-oxygen system is given in Ref. 13.

Solution Techniques

In this section we develop small perturbation solutions and also numerical solutions

for the system of equations derived for the 2V-port model.

Perturbation Solutions

We determine perturbation solutions around a nominal operating point of the rocket for the gas dynamic system of equations described in the previous paragraphs. These perturbation solutions will be used to obtain a transfer function representing the linear response of the gas dynamic system. The transfer function uses the oxidizer mass flow rate as the input and the system pressure for the output. The details of the derivation are shown in Ref. 13.

The transfer function specifically between the oxidizer mass flux input and the pre-combustion chamber pressure output can be written as

$$\Theta_1 = \frac{RT_1 \{ \Delta + [(F_{TC} - a_\bullet) + 2b_\bullet (L_2 s + e)] \tanh(\Delta L/2) \} I}{[(L_1 + L_2) s + e] \Delta + [L_1 (F_{TC} - a_\bullet) + 2] s + [2L_1 b_\bullet s - (F_{TC} - a_\bullet)] (L_2 s + e) \tanh(\Delta L/2)} \quad (A.14)$$

This function also represents the overall nature of the response for gas dynamic system. Similar expressions for the other variables can be also derived, but for the purpose of our arguments this expression is adequate.

Numerical Simulations

In order to check the validity of the assumptions used in the perturbation solutions, we also perform numerical simulations on the 2V-port model described in the previous section. We start with the finite differencing of the special derivative terms and use the second order central differencing over a uniform mesh along the axis of the port²⁵. For the time marching we use the Runge-Kutta fourth order method. The boundary conditions for the port equations are obtained from the pre and post combustion chamber dynamic equations. The numerical method used in the calculations gave stable and accurate results for a typical time step of 3×10^{-4} seconds and 80 mesh points in the port. The details of the numerical scheme is discussed in Ref 13.

The simulation code is applied to various transient events such as the throttling and the sinusoidal oxidizer flow rate case. Some of the results are discussed in the following section.

Discussion of Results for Pure Gas Dynamics

The response of the gas dynamic system for a hybrid motor can be deduced from the gas dynamic system transfer function. For example, the stability features of the system are revealed by the roots of the denominator of the transfer function which is the characteristic equation. However any investigation of the gas dynamic system response

requires an input for the combustion response that is represented by the F_{TC} term.

In this section we use the simplest quasi-steady combustion model. Namely, all the transient effects including the thermal lags in the solid and also the boundary layer delays except the gas dynamics are ignored in this model. For that reason, this model which does not allow any dynamic coupling between the sub-systems of the rocket is highly desirable to isolate the investigations to purely gas dynamic phenomena. The combustion response function can easily be derived from the general regression rate law after linearization and Laplace transformation.

$$F_{TC} = a_1 \bar{G}^{n-1} \quad (A.15)$$

For the case of the quasi-steady combustion the gas dynamic system transfer function is plotted in Figure 13. The operational parameters of this motor are discussed in Ref. 13. The plot shows a pole on the negative real axis and a row of imaginary poles also in the negative real half plane. The real pole resembles the volumetric capacity of the chamber, whereas the imaginary poles represent the longitudinal acoustic nature of the cavity. Note that all the poles have negative real parts, namely the both acoustic and also the global response components decay in time. The conclusion is that the pure gas dynamic system is stable in nature.

In order to test the validity of the transfer function we consider the trivial case of no mass addition and no temperature or velocity gradients in the port. In this case the transfer function simplifies to

$$\Theta_1 = \lim_{(F_{TC} - a_\bullet) \rightarrow 0} \frac{RT_1 I}{[(L_1 + L_2) s + e] + \frac{2s}{(F_{TC} - a_\bullet)} \tanh[(F_{TC} - a_\bullet) L/2]} \quad (A.16)$$

After the application of the L'Hospital's rule one obtains the well known transfer function for a simple single volume system.

$$\Theta_1 = \frac{RT_c I}{(L_1 + L_2 + L) s + e} = \frac{(RT_c / \theta c^*)}{\tau_f s + 1} \quad (A.17)$$

Here $\tau_f = L^* / c^*$ is the characteristic filling time of the motor chamber. The other parameters are defined as $L^* = V / A_p$ and $c^* = \Gamma_c \sqrt{RT_c}$ which are commonly used in rocket literature. Note that the total volume of the motor combustion chamber can be written as $V = (L_1 + L_2 + L) A_p$. This is the transfer function generally used (especially in the industry) in hybrid dynamic modeling.

The preceding equation does not predict the filling/emptying time scale of a hybrid rocket accurately. For that reason we develop a better estimate by the following argument. In the region of the s plane that the pole for the filling/emptying process is expected to be located, the following approximations can be made.

$$a_\phi \cong \frac{\beta}{RT_{acc}}, \quad b_\phi \cong 0, \quad \Delta \cong |a_\phi - F_{TC}| \quad (A.18)$$

Under these approximations the gas dynamic transfer function simplifies to

$$\Theta_1 = \frac{RT_c \left\{ 1 + \tanh \left(F_{TC} - a_\phi \right) L/2 \right\} I}{\left\{ L_1 + L_2 + \left[L_1 - L_2 + 2 / (F_{TC} - a_\phi) \right] \tanh \left(F_{TC} - a_\phi \right) L/2 \right\} s + e \left[1 - \tanh \left(F_{TC} - a_\phi \right) L/2 \right]} \quad (A.19)$$

Note that since $F_{TC} - a_\phi$ term is independent of s , this transfer function represent the behavior of a first order system with a characteristic time of

$$\tau_f = \frac{L_1 + L_2 + \left[L_1 - L_2 + 2 / (F_{TC} - a_\phi) \right] \tanh \left(F_{TC} - a_\phi \right) L/2}{e \left[1 - \tanh \left(F_{TC} - a_\phi \right) L/2 \right]} \quad (A.20)$$

The throttling response for a throttling ratio of 1.2 of the motor with the same specifications is calculated with different methods and plotted in Figure 14. Figure shows the time variation of the non-dimensional pre-combustion chamber pressure defined as $(P_1(t) - P_1^f) / (P_1^i - P_1^f)$ where P_1^i is the initial pressure and P_1^f is the final pressure. The striking observation is that the perturbation solution with the filling time defined by equation A.20 is in excellent agreement with the numerical simulation result. This shows that equation A.20 gives a good estimate for the filling time of the motor chamber within the context of the 2V-port model. The response according to the simple transfer function is also shown in Figure 14. It is obvious from the figure that the simple filling time argument for the hybrid combustion chamber is not valid.

Another observation from Figure 14 is that the numerical simulation result shows oscillatory behavior in the early stages of the throttling event. This oscillatory behavior is due to the excitation of the acoustic modes of the cavity by the disturbance caused by the sudden increase in the oxidizer mass flow rate. The oscillation frequency and the damping time scale of this mode of motion is consistent with the longitudinal acoustic mode findings for the same motor.

This kind of behavior is commonly observed in AMROC DM-01 motors⁴. Namely, a nonlinear disturbance causes the well damped oscillation of the chamber pressure at the longitudinal acoustic frequencies. The exciting disturbances in the actual tests are believed to be caused by: 1) a sudden blockage of the nozzle throat area by some disintegrated fuel fractions, 2) the ignition of a bulk of fuel oxidizer mixture

captured in the pre-combustion chamber or 3) the low frequency oscillations of chamber pressure.

We compare the predictions of our gas dynamic model for the acoustic frequencies of the chamber with the AMROC DM-01 motor test data. Table 7 shows the first five longitudinal frequencies determined by the perturbation solutions. Since our model is isothermal, these frequencies are corrected by multiplying with the factor of $\sqrt{\gamma}$. An average γ value of 1.2 is used for the calculations. Note that the corrected frequencies falls in the range of observed frequencies for every calculated acoustic mode.

The gas dynamic code developed to perform simulations on the 2V-Port model is used to confirm the findings of the perturbation solutions on the frequency response of the system. In the simulations, sinusoidal oxidizer mass flow rate inputs at various frequencies are applied as the forcing function and the phase and the amplitude of the chamber pressure oscillations are determined. The agreement between the perturbation solutions and the numerical simulations for the frequency response of the system is also found to be satisfactory.

The simulation code is also used to investigate the response of the gas dynamic system to a disturbance. The oxidizer mass flow input is

suddenly increased 10% above its operating value and held at its elevated level for 0.03 seconds before its suddenly reduced to its original value. The pressure time history for this simulation is shown Figure 15. It is clear that the disturbance excites the acoustic modes of the chamber similar to the previous observations. These excited modes decay in a moderate time scale as it was observed in the AMROC tests. A similar simulation is performed to determine the effect of nozzle blockage on the stability by reducing the nozzle throat area for a brief period of time. The results for the nozzle area perturbation case are identical in nature to the results of the oxidizer mass flow rate perturbation. In short, a nonlinear disturbance causes the ringing of the hybrid chamber which decays in time.

Finally a simple delay (τ_{MH}) on the combustion response is applied to the gas dynamic system.

$$F_{TC} = a_1 n \bar{G}^{n-1} e^{-\tau_{MH}} \quad (A.21)$$

Both the perturbation solutions and also the numerical simulations indicate that any positive combustion delay have negligible effect on the stability nature of the system.

15) Tables and Figures:

Physical Phenomenon:	Time Scale:	Explanation:
1) Solid phase kinetic times	$\tau_{sp} < 10^{-3}$ sec	Degradation mechanisms of the polymer
2) Gas phase kinetic times	$\tau_{gp} < 10^{-3}$ sec	Hydrocarbon combustion mechanisms
3) Feed system response times	(Varies greatly from system to system)	Response time of the feed system
4) Evaporation times	$\tau_{evap} = f(U_o, T_1, \Delta P)$	Evaporation process of the liquid oxidizer
5) Thermal lags in solid	$\tau_H \propto \kappa / \dot{r}^2 \approx 10^{-1}$ sec	Thermal profile changes in the solid grain
6) Boundary layer diffusion times	$\tau_{bl} \propto L / u_c \approx 10^{-1}$ sec (Varies greatly from case to case)	Turbulent boundary layer diffusion processes
3) Acoustic times (longitudinal)	$\tau_a \propto L / c \approx 10^{-3}$ sec (Varies greatly from case to case)	Propagation of the acoustic waves
7) Gas dynamic filling times	$\tau_{fH} \propto L^2 / c^* \approx 10^{-1}$ sec (Varies greatly from case to case)	Global mass flow balance

Table 1: The transient time scales of various phenomena in a typical hybrid rocket. Only the last four are modeled in this paper.

AMROC Motor (thrust)	O/F	$\dot{c}_{\text{theo}}$ (lb/sec)	$\dot{c}_{\text{exp}}$ (lb/sec)	L^* (in)	f (Hz)	τ_{W2} (msec)
S Motor (10k)	1.45	5584	5361	570	11	48.9
Half-Scale (33k)	1.45	5584	5361	1305-1430	4.8	117.4
H-500 (75k)	1.45	5584	5361	1770-2340	4.0	176.3
DM-01 (250k)	1.55	5700	5472	2168	2-3.5	204.8

Table 2: Summary of parameters used in the frequency estimations for AMROC motors.

Test No.	$\bar{D}$ (in)	$\dot{m}_o$ (lb/sec)	G_o (lb/in ² -sec)	G_i (lb/in ² -sec)	P_c (psi)	O/F	f (Hz)	ΔP_{osc} (psi)	τ_{W2} (msec)
1 (GOX)	4.25	6.0	0.423	0.571	600	2.85	13	200	48.8
2 (GOX)	4.25	5.7	0.402	0.541	550	2.89	8	300	47.2
7 (GOX)	4.20	4.7	0.339	0.458	440	2.89	15	100	44.7
8 (GOX)	4.22	5.1	0.365	0.492	260/280	2.87	20	400	25.5
1 (LOX)	-	-	0.222	0.31	435	2.51	8	250	66.2

Table 3: Summary of parameters used in the frequency estimations for the 11 inch Hybrid motor tests. Grain length is 102 inches.

Test No.	$\bar{D}$ (in)	L (in)	$\dot{m}_o$ (lb/sec)	G_o (lb/in ² -sec)	G_i (lb/in ² -sec)	P_c (psi)	O/F	f (Hz)	ΔP_{osc} (%)	τ_{W2} (msec)
3	3.4	102	3.44	0.38	0.527	745	2.6	6-10	15	66.5
4	4.3	102	3.36	0.23	0.325	325	2.4	10-20	20	47.4
6	3.7	102	8.36	0.78	1.024	750	3.2	10-15	12	33.7
7	5.3	102	3.41	0.15	0.22	335	2.1	6-15	15	73.3
8	5.7	102	3.87	0.15	0.209	850	2.6	2-5	60	191.7
9	6.1	102	6.34	0.28	0.392	215	2.6	6-20	33	25.9
15	5.4	108	10.20	0.45	0.599	1025	3.0	4	10	83.9
10	6.1	102	5.93	0.20	0.278	215	2.6	10-25	5	36.5

Table 4: Summary of parameters used in the frequency estimations for the NASA/MSFC 11 inch Hybrid motor tests.

Test No.	$\bar{D}$ (in)	L (in)	$\dot{m}_o$ (lb/sec)	G_o (lb/in ² -sec)	P_c (psi)	O/F	f (Hz)	τ_{M2} (msec)
1	2.0	27	3.44	0.29	500	4.0	19	20.6

Table 5: Summary of parameters used in the frequency estimations for the Arizona State Hybrid motor test.

Test No.	L (in)	G_o (lb/in ² -sec)	P_c (psi)	O/F	f (Hz)	τ_{M2} (msec)
4F-4	32	0.44	528.0	3.97	41.6	13.5
4F-5	32	0.49	551.0	3.59	39.9	12.6
4F-1b	32	0.20	561.0	2.72	14.4	29.7
4F1-c	32	0.16	542.0	3.06	13.6	36.1
4P-01	45	0.39	318.0	2.69	40.8	12.3
4P-02	45	0.38	993.8	2.48	13.2	38.5
4P-03	45	0.40	939.1	2.65	14.4	35.6
4L-03	45	0.31	641.6	2.69	12.7	30.9
4L-04	45	0.52	656.7	2.66	23.6	18.9
4L-05	45	0.46	649.3	2.72	19.1	21.2
4L-08	45	0.44	525.0	2.64	23.3	17.9
4L-01	45	0.38	318.7	2.40	39.6	12.4
4P-04	45	0.21	159.1	1.73	42.4	10.7
4L-09	45	0.26	265.3	1.54	20.3	13.7
4L-10	45	0.43	590.0	2.89	27.0	18.2
4L-11	45	0.11	213.0	1.56	17.8	27.2
4L-12	45	0.13	301.0	2.01	15.54	32.2
4NF-01	45	0.39	602.2	2.77	22.60	23.1
4NF-02	45	0.22	600.4	2.34	11.10	40.4
4NF-03	45	0.36	500.8	2.96	23.20	21.4
4NF-04	45	0.48	568.5	3.01	32.75	18.0
4L-14	45	0.43	524.5	2.51	25.15	18.0
4ST-02	45	0.45	540.0	2.71	24.86	18.2
4L-15	45	0.31	555.5	2.09	14.94	26.2
4Rep-02	45	0.45	402.3	2.63	36.49	13.5

Table 6: Summary of parameters used in the frequency estimations for the NASA Ames Hybrid motor tests.

Longitudinal Acoustic Mode	Estimated (Isothermal) Frequency (Hz)	Corrected Frequency (Hz)	Observed Frequency (Hz)	Estimated Amplification (1/sec)
1 st	48.4	53.0	50-55	-22
2 nd	96.7	103.9	100-110	-22
3 rd	142.6	156.1	150-165	-21
4 th	191.1	209.3	200-220	-19
5 th	240.7	263.6	250-275	-18

Table 7: The estimated and observed longitudinal acoustic frequencies for AMROC DM-01 motor.

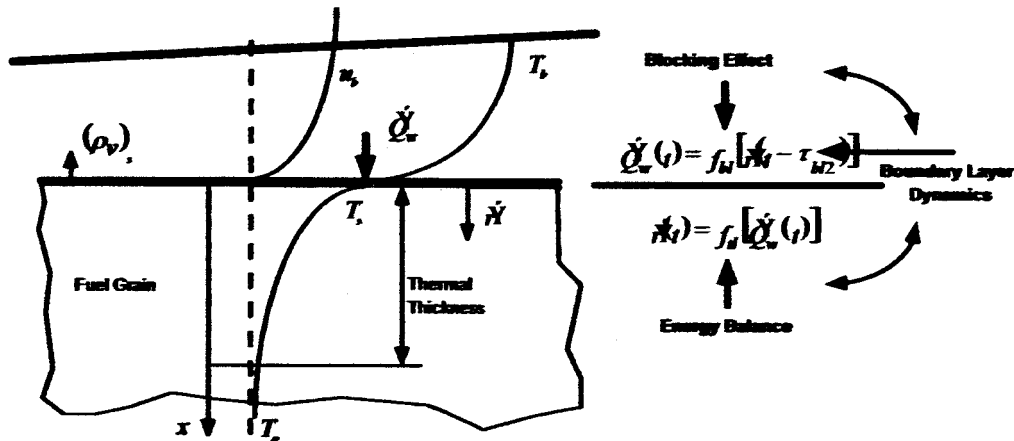


Figure 1: Thermal lag model and TC coupling mechanism.

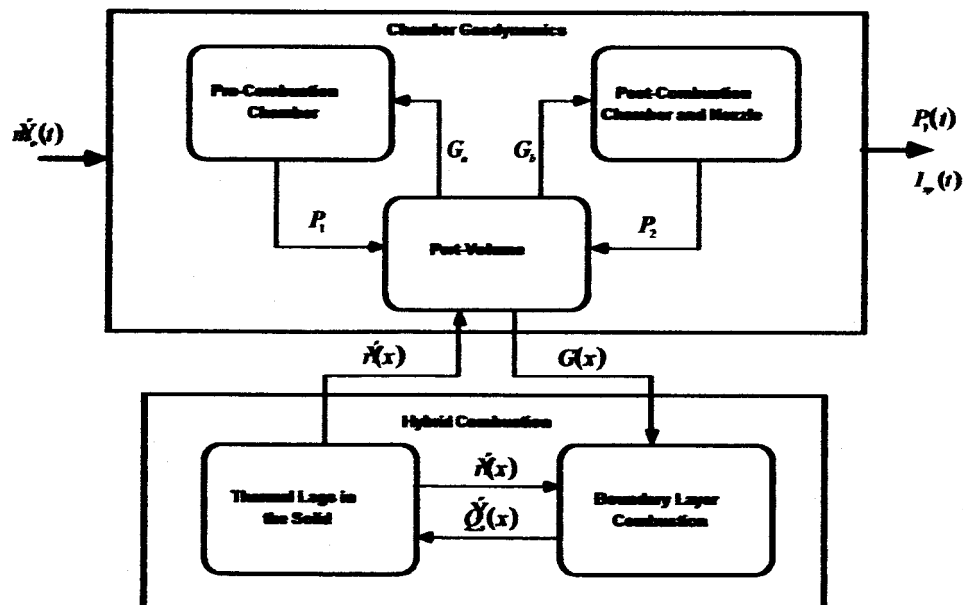


Figure 2. The schematic of the TCG coupled system.

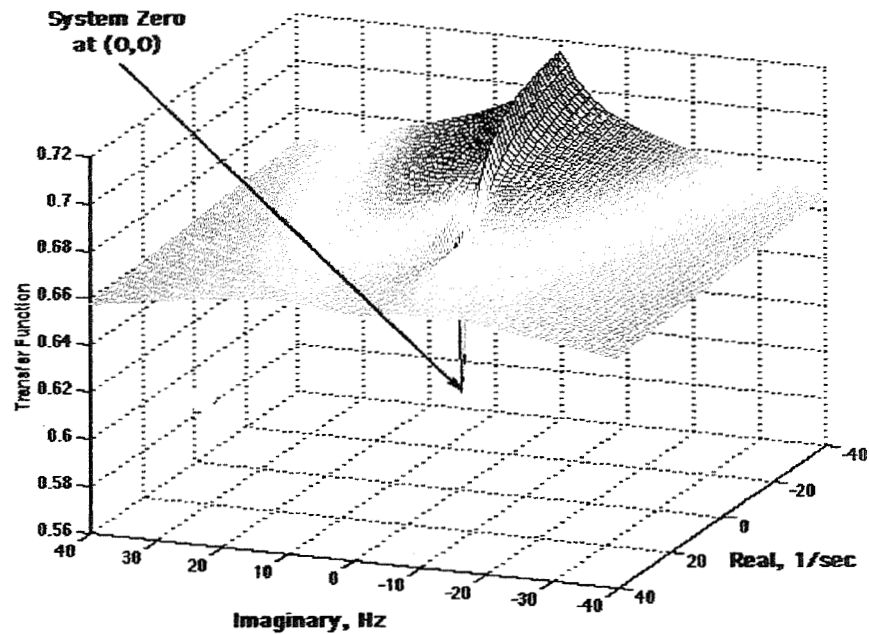


Figure 3: Plot of the transfer function of the TC coupled system with no delays ($\tau_{b1} = \tau_{b2} = 0$). This plot is for a HTPB system with $E_a = 15$ kcal/mole.

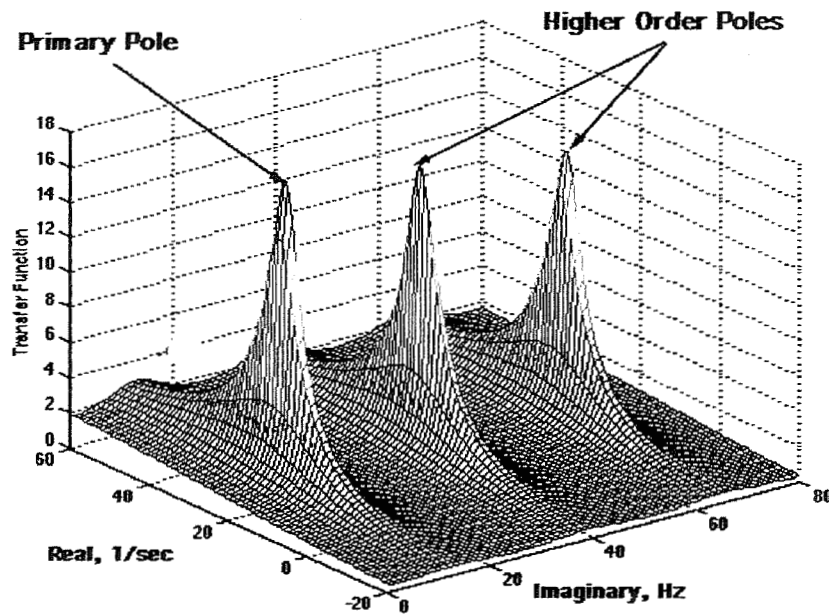


Figure 4: The unstable poles of the TC coupled system in the s plane for $\tau_{b1} = 0$, $\tau_{b2} = 38$ msec. This plot is for a HTPB system with $E_a = 15$ kcal/mole.

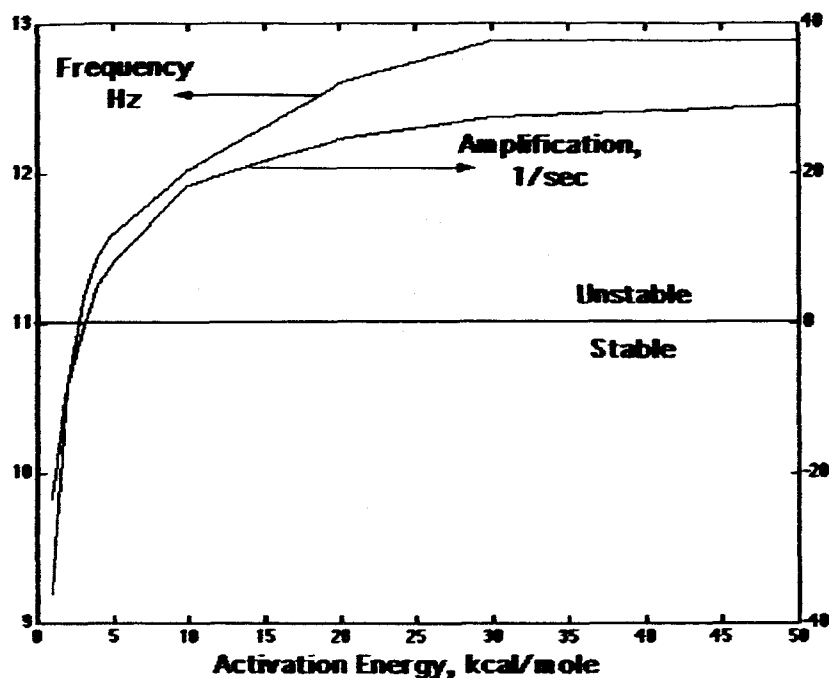


Figure 5: Effect of activation energy on the oscillation frequency and amplification.

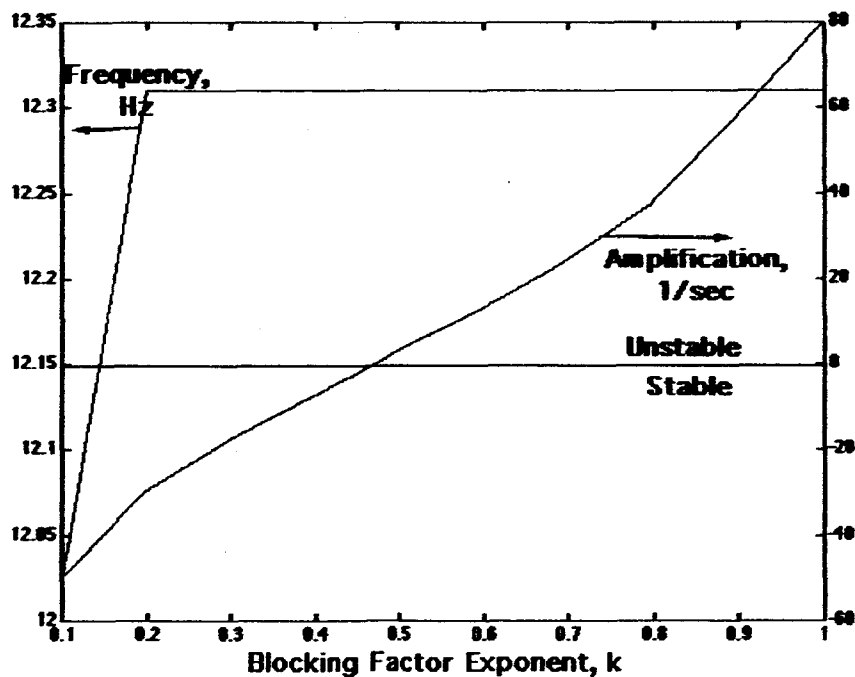


Figure 6: Effect of blocking factor exponent on the oscillation frequency and amplification.

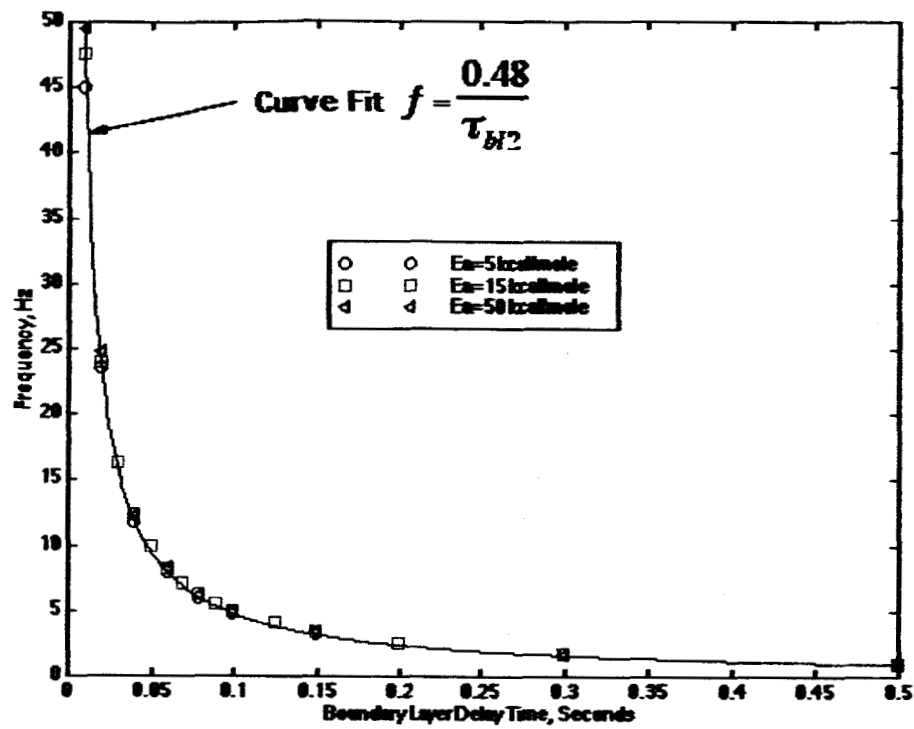


Figure 7: Effect of boundary layer delay time on the oscillation frequency for various activation energies.

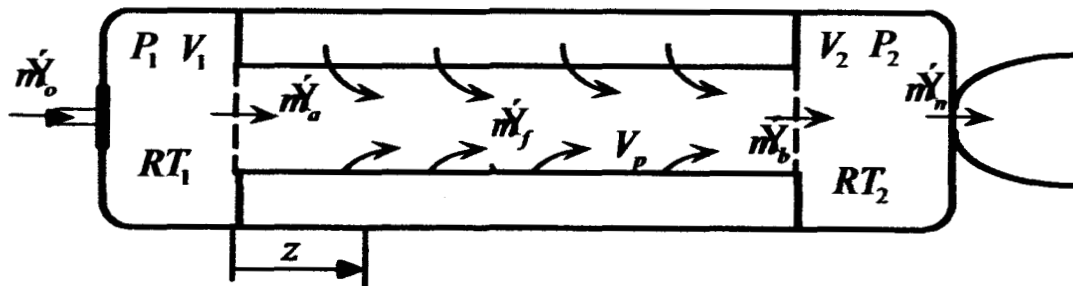


Figure 8: The schematic of the 2V-port model.

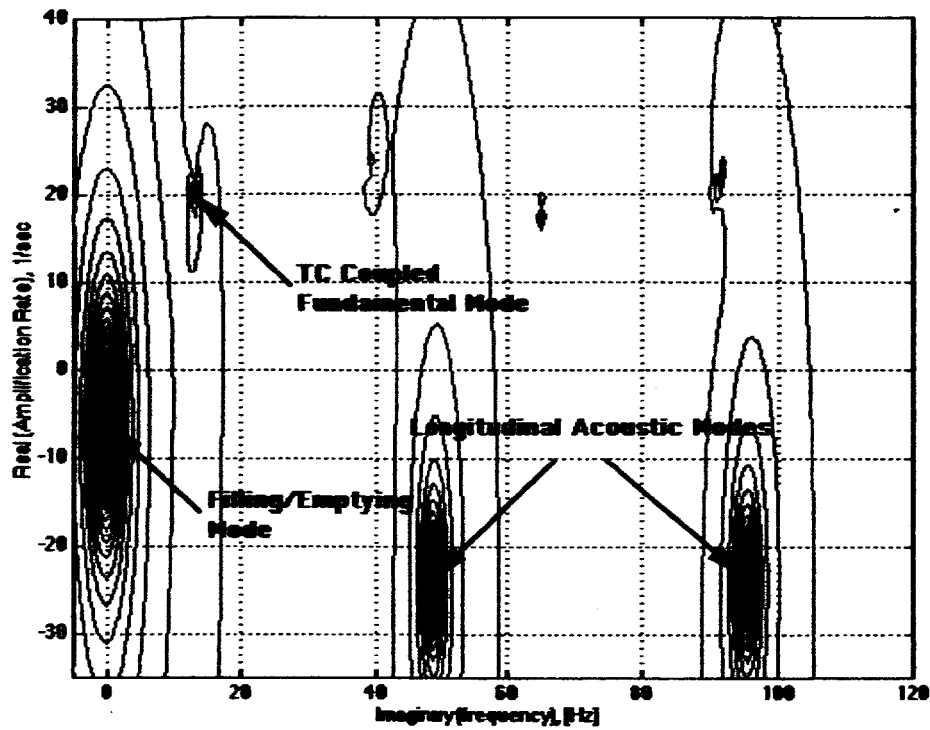


Figure 9: The contour plot of the TCG coupled system with $\tau_{b12} = 38$ msec. Most of the transient features of a hybrid system can be deduced from the locations of the poles.

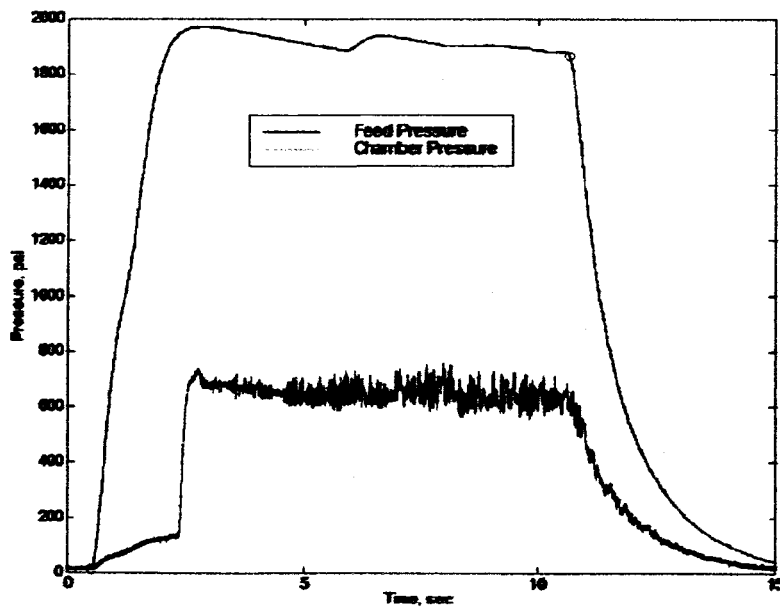


Figure 10: Chamber and feed pressure time traces for the paraffin-based motor test, 4L-05.

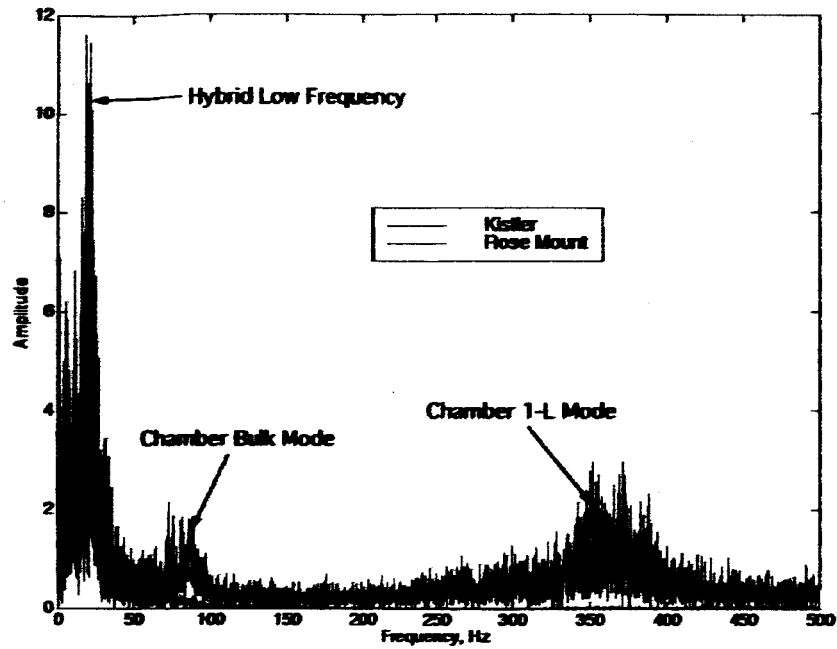


Figure 11: Fourier transform of the chamber pressure for the paraffin-based motor test, 4L-05.

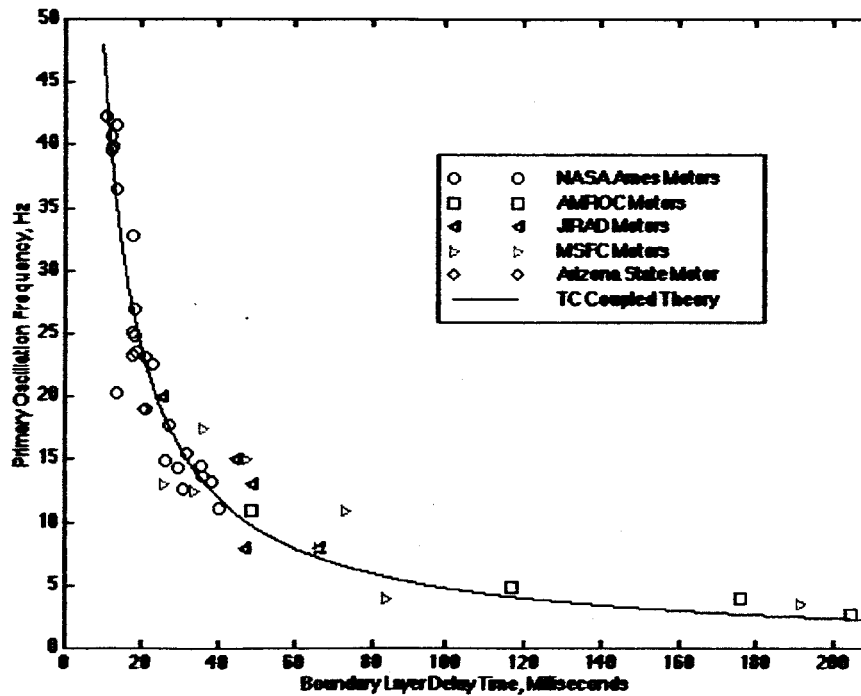


Figure 12: Comparison of the TCG coupled oscillation frequency prediction with the hybrid motor test data available in the literature.

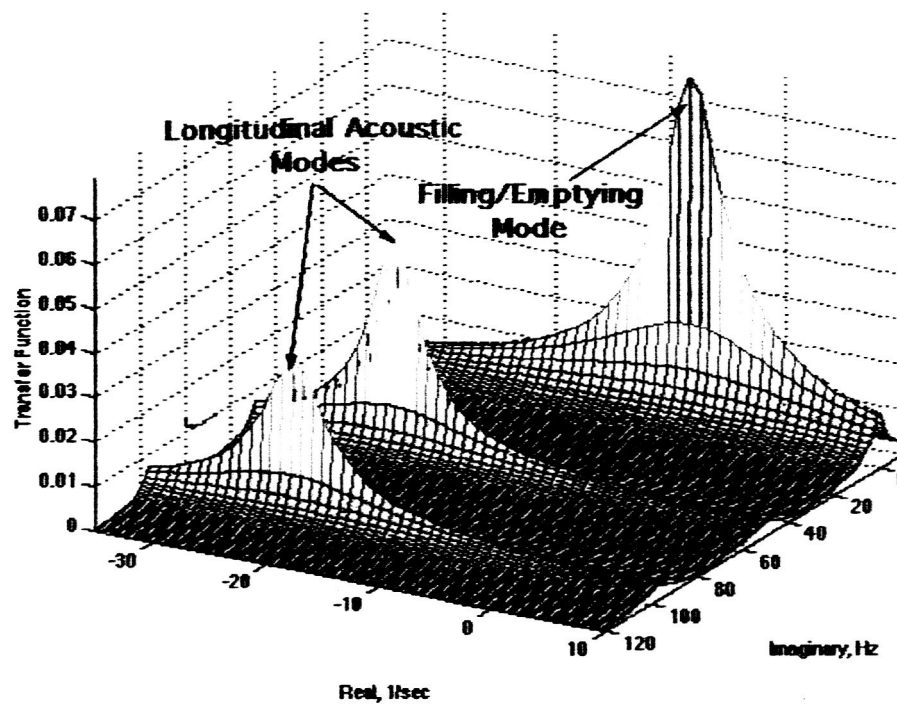


Figure 13: The plot of the transfer function for the pure gas dynamic system.

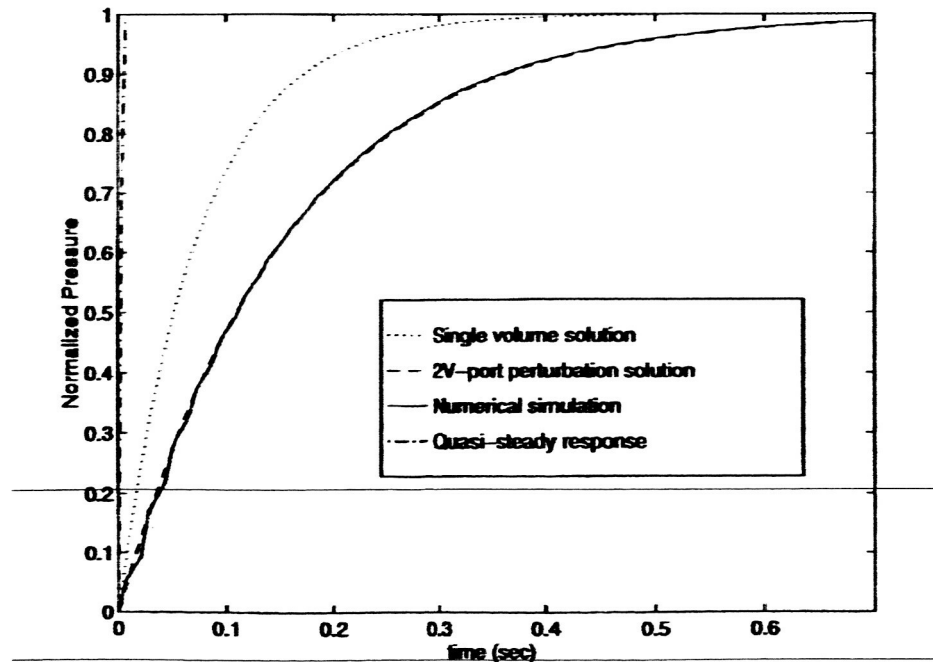


Figure 14: The throttling response for the pure gas dynamic system.

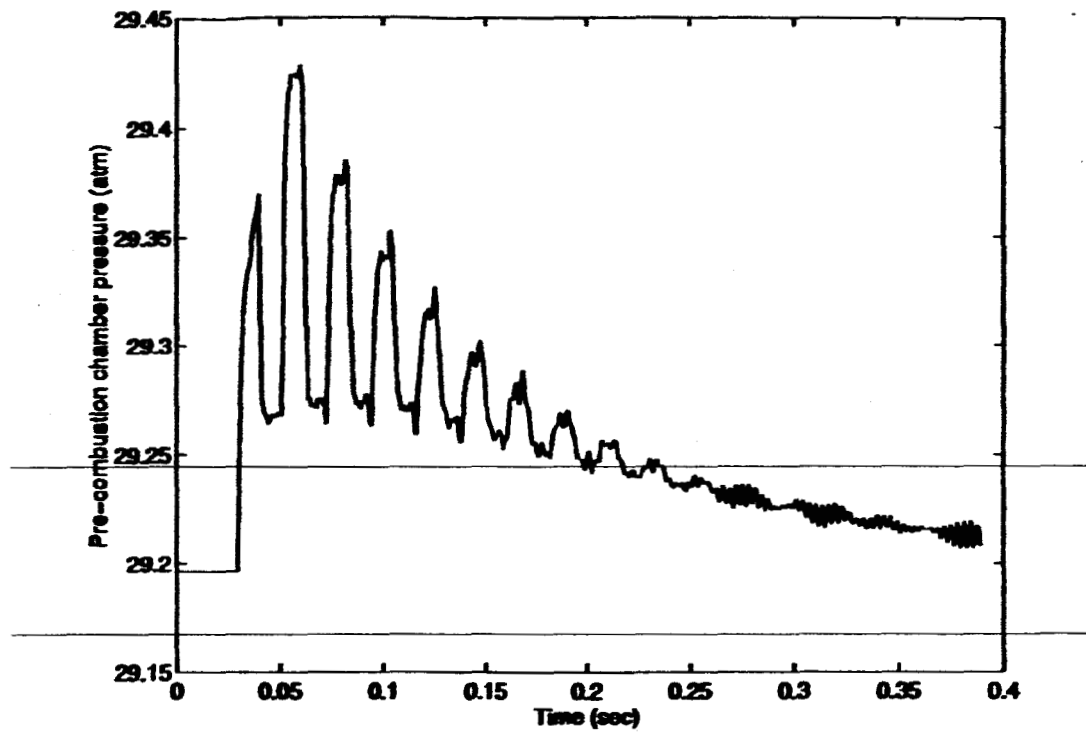


Figure 15: The response of the pre-combustion chamber pressure to a disturbance of the oxidizer mass flow rate.